Introduction to Measurement & Calculation

Significant Figures

Significant figures indicate the *reliably known digits* in an experimental value; they are typically *determined by the precision of the instrument(s) used to make the measurement(s)*. When performing calculations using measured values, keep in mind that not all digits in the answer displayed by your calculator are necessarily significant (*see* **Calculation & Significant Figures**).

In the lab we observe the following (standard) conventions regarding significant figures:

1. All non-zero digits ARE significant

2. Zeroes *between* non-zero digits ARE significant

3. *Leading zeroes* are NOT significant

4. *Trailing zeroes* ARE significant *IF* there is a decimal point

5. All digits ARE significant when a number is expressed in *scientific notation*

(Example) *The numbers 19.7, 80, 0.0031 and 0.2050 have 3, 1, 2 and 4 significant figures respectively. As written, 15,600 would have 3 sig figs; if written as 15,600. (with an explicit decimal point) it would be considered to have 5 sig figs. One could avoid potential confusion by using scientific notation, i.e. writing 1.56×10^4 for 3 sig figs or 1.5600×10^4 for 5 sig figs.*

Precision and Accuracy

Precision and **accuracy** are terms which are often (*incorrectly!*) used interchangeably.

Precision refers to how well a value is known. There is no such thing as an exact measurement; there is *always* some error or uncertainty associated with it. The uncertainty in a measurement may be very small, but it is never zero. The uncertainty describes the range of values within which you are reasonably sure the measurement lies. The smaller the uncertainty, the more precisely the value is known. Precision may be indicated by the number of decimal places in a value.

(Example) *The acceleration due to gravity is found by various experimental methods to be 9.8, 9.81 and 9.805 m/s^2. The third value, 9.805 m/s^2, is the most precise because it is determined to within one thousandth of a m/s^2 (3 decimal places). The first value, 9.8 m/s^2, is the least precise because it is only determined to one tenth of a m/s^2 (1 decimal place).*

Accuracy refers to how close a result is to the actual or true value.

(Example) *9.7 m/s^2 is a more accurate value of the acceleration due to gravity than 9.6 m/s^2 is, since the expected value is 9.8 m/s^2 (to two significant figures). The values are equally precise, however, since they are both determined to a tenth of a m/s^2 (1 decimal place).*

Recording of Experimental Values

In the physics lab experimental values will be recorded to the precision of the measuring instrument, which is taken to be the smallest division indicated on the instrument. This must however be tempered with reasonable judgment as cases may arise where the precision of the measurement is much less than the (theoretical) precision of the measuring instrument.

It is good experimental practice to record all measurements *as read directly from an instrument or measuring device*, without any unit conversion or calculation. For example, if a scale used to measure mass reads in grams, then record any measurements made with that instrument in grams. If necessary, convert to kilograms or other units only *after* the measurement is made and recorded in grams.
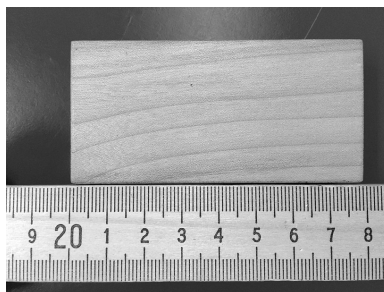
Metre Stick (metric scale or metric ruler)

Figure A: measurement of a rectangular block

The familiar metre stick is usually divided into centimetres and tenths of centimetres (millimetres), with the centimetres being numbered - for example, see Figure A. The smallest divisions are tenths of a cm (1 mm) and therefore the precision of the metre stick is 1 mm or 0.1 cm. If the ruler is transparent and the graduations are on the underside, it may be laid flat on the object. If the ruler is not transparent, or if the graduations are on the upper surface, the ruler should be held on edge so that the graduations are in contact with the object. This eliminates errors due to parallax.

A linear distance measured with a ruler consists of two readings, which are the positions of both ends of the object. Typically, the "zero" end of a metre stick is in poor condition and is *not* used as one of the positions; otherwise, any other convenient line on the metre stick may be used. In one step, a complete length measurement would be recorded as:

 Length = (position of one end of block) - (position of other end of block) = (length of block)

(Example) *The measurement of the length of the block in Figure A would be recorded as:*

$$Length \quad = \quad 27.9 \ cm - 20.0 \ cm \ = \ 7.9 \ cm$$

Lengths may be measured *directly* if a (plastic) ruler has a well defined "zero" (i.e. using 0.0 cm).
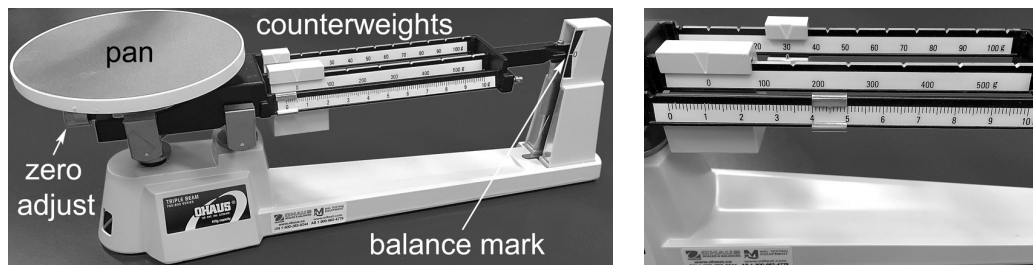
Triple Beam Balance

Figure B: Triple beam balance (left); closeup of counterweights when measuring (right).

The triple-beam balance derives its name from the three beams which carry the counterweights that balance the weight on the pan. The balance determines the mass m of an object by exploiting the linear relationship $W = mg$ between an object's weight W & the gravitational acceleration g; this allows the balance to be graduated directly in mass units (e.g. readings yield mass in grams).

The object whose mass is to be determined is placed on the pan and a balance condition is achieved by moving, *in order*, the counterweights on the hundreds beam, the tens beam and the units beam. A static balance condition is indicated by the pointer on the beam being lined up with the central mark on the scale on the end frame. The hundreds and tens beams have notches in which the counterweights *must* sit in order for them to be in the correct positions. The units counterweight may be placed at any position along its beam, the position being shown by the pointer in the centre of the counterweight. The smallest division on the units beam is 0.1 g and therefore the precision of the triple-beam balance is ± 0.1 g, *i.e. to the nearest tenth of a gram*.

The balance is subject to a zero error, i.e. when all the counterweights are set at 0, the pointer may not be lined up with the central mark on the frame. The zero point is adjusted by turning the adjustment knob beneath the pan. The zero reading *must* be properly set BEFORE using the balance.

(Example) *Assuming proper zeroing, the balance reading in Figure B is* $30.0 + 0.0 + 4.5 = 34.5$ *g.*

Calculation & Significant Figures

1). When **adding or subtracting** numbers, the precision of the sum or difference is the same as that of the *least precise number* in the set.

(Example) $4.513 + 3.9 = 8.413$ *is rounded to 8.4 because 3.9 is only good to one decimal place.*

(Example) $110 + 39.5 = 149.5 = 150$ *since 110 (*$=1.1 \times 10^2$*) is only good to the 'tens' place.*

2). When **multiplying or dividing** numbers, the number of significant figures in the product or quotient is the same as that of the number in the set which has the *fewest significant figures*. **Note:** multiplication or division by a constant does *not* affect the determination of significant figures.

(Example) $\dfrac{4.513 \times 7.8}{2.001} = 17.5919$ *is rounded to 18 because 7.8 has only two significant figures.*

3). When dealing with **mixed calculations** (e.g. combinations of addition, subtraction, multiplication and division one calculation) it is *necessary* to perform the calculation in steps, obeying *mathematical order of operations*, in order to determine the proper significant figures in the final answer. ***You CANNOT do such calculations in a single step on your calculator and properly account for significant figures.*** To **minimize round-off error** *it is common practice to retain (at least) one EXTRA digit* during steps in a calculation *and mark the position of the least significant digit by underlining it, e.g. 7.02$\underline{3}$6 or 591.$\underline{4}$22 where the final 6 or 22 to the right of the underlined digit are NOT significant.*

(Example) *Calculate the volume of a rectangular solid if the length, width and height are measured to be 16.0 cm, 7.95 cm and 3.9 cm, respectively.*

$$V = L \times W \times H = (16.0)(7.95)(3.9) = 4\underline{9}6.1 = 5.0 \times 10^2 \; cm^3$$

Since all of the operations are multiplication/division, we can do this calculation in one step, applying the rule for sig figs & multiplication/division. In this case our final answer should have two sig figs, since that is the lowest number of sig figs in the product; note that this results in rounding UP to 500 and that scientific notation is required to clarify that only two digits are significant.

(Example) *Calculate the density of the solid (above) if its mass is measured as 280.9 g.*

$$\rho = \frac{m}{V} = \frac{280.9}{4\underline{9}6.1} = 0.5\underline{6}6 = 0.57 \; g/cm^3$$

*Since all of the operations are multiplication/division, we can do this calculation in one step, applying the rule for sig figs & multiplication/division. In this case we use the **unrounded** value for the volume while noting the least significant digit for the purposes of rounding our final answer.*

(Example) *Calculate the surface area of a rectangular solid if the length, width and height are measured to be 16.0 cm, 7.95 cm and 3.9 cm, respectively.*

$$
\begin{aligned}
\text{Area} &= 2LW + 2LH + 2WH \\
&= 2(16.0)(7.95) + 2(16.0)(3.9) + 2(7.95)(3.9) \\
&= 25\underline{4}.4 + 12\underline{4}.8 + 6\underline{2}.01 \\
&= 4\underline{4}1.21 \\
\text{Area} &= 440 \; cm^2 \; or \; 4.4 \times 10^2 \; cm^2
\end{aligned}
$$

Using order of operations, the multiplications are performed FIRST. We apply the rule for sig figs & multiplication/division to EACH of the three terms; as each term contains a number with a minimum of two sig figs, this determines what we can retain for the product. Note that we ignore the impact on sig figs of the constant '2' in each term and keep an extra digit (or two) after the underlined significant digit for each term. NOW the addition is performed and we apply the rule for sig figs & addition/subtraction; in this case, our least precise value in the sum is known only to the 'tens' and so we round our final answer appropriately and add units.

*IF the area were used in a later calculation we would utilize the **unrounded** value but underline the least significant digit for the purposes of applying the sig fig rules during that calculation.*

Comparing Values

1). **Percent difference** compares two *rounded* experimental measurements:

$$\text{Percent Difference} \quad = \quad \frac{(higher\ value - lower\ value)}{average\ value} \times 100\%$$

Percent difference is **always positive** and **is reported with at most 2 significant figures**.

(Example) *Two students measure a rectangular block and calculate its volume. One finds the volume to be 27 cm^3 while the other gets 28 cm^3. The percent difference between their results is*

$$
\begin{aligned}
\text{Percent Difference} \quad &= \quad \frac{(high - low)}{average} \times 100 \\
&= \quad \frac{(28 - 27)}{27.5} \times 100 \quad = \quad \frac{1}{27.5} \times 100 \quad = \quad 3.636\% \quad = \quad 4\%
\end{aligned}
$$

Rounded for significant figures, the two values differ by 4%.

2). **Percent deviation** compares a *rounded* experimental measurement with its expected value[1]:

$$\text{Percent Deviation} \quad = \quad \frac{(experimental - expected)}{expected} \times 100\%$$

Percent deviation may be **positive or negative** (i.e. whether the rounded experimental value is greater than or less than the expected value) and **is reported with at most 2 significant figures**.

Percent deviation is sometimes also referred to as **percent error**.

(Example) *A student calculates the density of aluminum to be 2.5 g/cm^3; the published or expected value is 2.70 g/cm^3. The percent deviation is*

$$
\begin{aligned}
\text{Percent Deviation} \quad &= \quad \frac{(experimental - expected)}{expected} \times 100 \\
&= \quad \frac{(2.5 - 2.70)}{2.70} \times 100 \quad = \quad \frac{-0.20}{2.70} \times 100 \quad = \quad -7.407\% \quad = \quad -7\%
\end{aligned}
$$

Rounded for significant figures, the experimental value is 7% BELOW the expected value.

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[1]The *expected value* may be an accepted value for a physical constant like those given in a handbook of constants (e.g. acceleration due to gravity at Earth's surface) or something calculated theoretically using values determined outside of the lab (e.g. using numbers provided by the manufacturer or accepted values for physical constants).

Laboratory 1 Pre-Lab (value: *2 marks*)

Submit to your lab instructor *ALONG WITH YOUR LAB* at the *END* of the lab period.

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1. Which of the following values is the most *precise*: 9.891m, 989.10m, 98910.0m? Why?

2. How many significant figures are there in (a) 2.005 (b) 1001 (c) 0.00380 (d) 1.6×10^4 (e) 300?

3. Perform the following calculations, using the rules given in this lab for significant figures:
(a) $4.1 + 2.95$ (b) 8.12 - 1 (c) $272/2.2$ (d) 2.005×8.9 (e) $(10. + 4.2)/1.2$

4. Determine (showing all work) the measured (a) length and (b) mass in the images below.
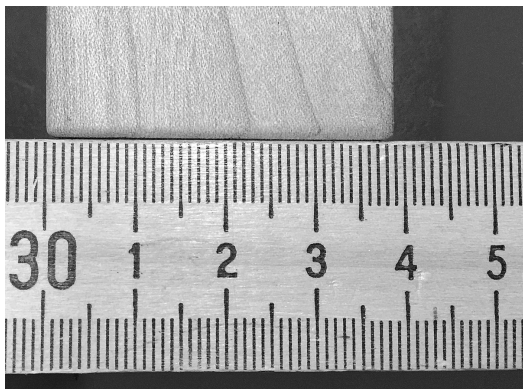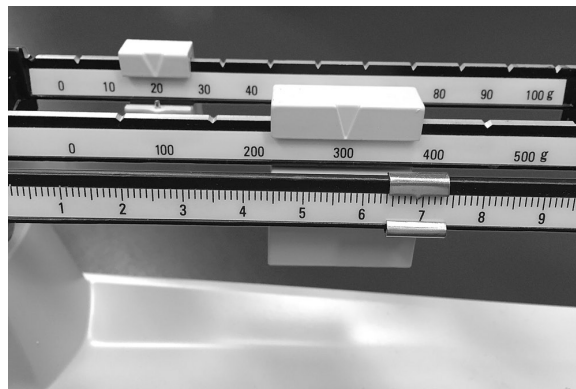
(a)   (b)

5. A block's volume is determined twice (experimentally) and found to be 27cm^3 and 28cm^3, respectively. *Compare* the volumes with each other (use *percent difference* & round for sig figs).

Laboratory 1:
Measurement, Precision & Scaling

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Metre stick, metre stick callipers (2), triple-beam balance, small & large wooden blocks, large white bucket filled with tennis balls, tennis ball 'track' (holder).

OBJECTIVE

1. To learn how to properly use a ruler and a triple beam balance to make measurements.

2. To become familiar with *experimental uncertainty* as it applies to such measurements.

3. To practice working with *significant figures* in measurements & during calculations.

THEORY

In this experiment, the linear dimensions and the masses of two similar rectangular blocks are measured. The volumes, surface areas and densities of the blocks are then determined. The ratios of the linear dimensions, the surface areas and the volumes of the two blocks are compared in order to determine the relationship, if any, between the two blocks.
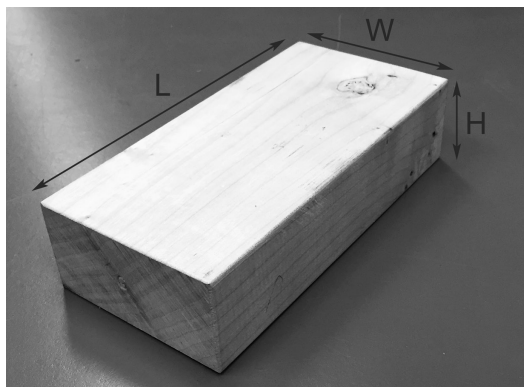

Figure 1: dimensions for a rectangular block

Volume

A *rectangular block* of length L, width W and height H will have a volume given by

$$V = L \times W \times H \tag{1}$$

An example is shown in Figure 1, with the sides chosen such that $L > W > H$.

A *sphere* of radius R (or diameter $D = 2R$) will have a volume given by

$$V = \frac{4}{3}\pi R^3 = \frac{\pi D^3}{6} \tag{2}$$

A *cylinder* of radius R (or diameter $D = 2R$) and height H will have a volume given by

$$V = \text{Area of base} \times H = \pi R^2 \times H = \frac{\pi D^2 \times H}{4} \tag{3}$$

Density

The density ρ of an object is found by dividing its mass m by its volume V,

$$\rho = \frac{m}{V} \tag{4}$$

The units most commonly used for density are g/cm^3 or kg/m^3.

Surface area

A *rectangular block* of length L, width W and height H will have a surface area given by

$$A = 2LW + 2LH + 2WH \tag{5}$$

Laboratory 1: Measurement, Precision & Scaling

Part A: Measurements of the two blocks

1. Check that the balance reads zero when the weights are in their zero positions. Adjust if required.

2. [*1 mark*] Record the mass of the LARGE block: $m_{large} =$ ———————.

3. [*1 mark*] Choose some graduation on the metre stick (e.g. the 10.0 cm mark) as your zero reading. Using the definitions given in Figure 1, place the *length L* of the block against the metre stick with one end aligned with your selected zero position. Record both the zero position and the position of the other end of the block in the table below and calculate the length of the block using these values. Do this for BOTH (opposite) sides of the block. Repeat this process for the *width W* and *height H*. Use correct significant figures.

Large block dimensions (cm)			
	End position (cm)	Zero position (cm)	Calculated size (cm)
length (side 1)			
length (side 2)			
width (side 1)			
width (side 2)			
height (side 1)			
height (side 2)			

4. [*1 mark*] Record the mass of the SMALL block: $m_{small} =$ ———————.

5. [*1 mark*] Repeat the six (6) size measurements above for the SMALL block; measure the position of both ends & subtract to determine the size of a given side. You do NOT need to make a table (like for the large block, above) but do show your work (one line per side) & watch sig figs.

6. [**1 mark**] Why do we *need* to measure *both* sides of the blocks (i.e. 2 lengths, 2 widths, etc.)?

7. [**3 marks**] Calculate the *average* length, *average* width, and *average* height of the large & small blocks and then take the *ratio* (large/small) of your average dimensions; record all values in the table below. *Show a complete sample calculation for the middle column (width), including all equations used & utilizing correct significant figures* in the space provided BELOW the table.

Hint: perform calculations carefully (in steps) - unexpected results arise in a couple of the cases!

	average length L_{avg} (cm)	average width W_{avg} (cm)	average height H_{avg} (cm)
large block			
small block			
ratio (large/small), s			

Given the above, how is the large block related to the small one? Calculate the *average ratio*, s_{avg}.

8. Tidy the apparatus.

Part B: Volume and Density

1. [**2 marks**] Calculate the *volume* of each block using Equation 1 & their *average* dimensions. Show all calculations fully, including any equations used & utilizing correct significant figures.

2. [**2 marks**] Since *each* dimension of the large block is (approximately) s_{avg} times larger than the corresponding dimension of the small block, we'd expect the volume of the large block would be $s_{avg} \times s_{avg} \times s_{avg} = s_{avg}^3$ times larger than the volume of the small block. Using your (unrounded) volumes, calculate the ratio of the volumes, V_{large}/V_{small}. Then calculate s_{avg}^3 (using your s_{avg} from Part A) and *compare* to the ratio of the volumes using *percent difference. Comment.*

3. [*2 marks*] Calculate the *density* of each block using Equation 4 and their mass and (unrounded) volume. Show calculations fully, including all equations used & utilizing correct significant figures.

4. [*2 marks*] How *should* the two blocks' densities compare to each other? Why? Use *percent difference* to *compare* the densities of the two blocks and *comment*. Show all work fully.

Part C: Surface Area

5. [**2 marks**] Calculate the *surface area* of each block using Equation 5 & their *average* dimensions. Show all calculations fully, including equations used & utilizing correct significant figures.

6. [**2 marks**] Since *each* dimension of the large block is (approximately) s_{avg} times larger than the corresponding dimension of the small block, we'd expect any surface on the large block would be $s_{avg} \times s_{avg} = s_{avg}^2$ times larger than the corresponding surface on the small block. Using your (unrounded) surface areas, calculate the ratio A_{large}/A_{small}. Then calculate s_{avg}^2 (using your s_{avg} from Part A) and *compare* to the ratio of the surface areas using *percent difference. Comment.*

Part D: How many tennis balls fit in the bucket?

1. Slide the provided *metal calipers* onto the meter stick, vertical sides facing *inward*. Align the inner edge of one caliper with an easy to reference value (*e.g.* 10.0 cm) and lock it in place. The other caliper should be left loose enough to adjust its position easily without it sliding around.

2. [**1 mark**] Place a *single* tennis ball on the provided 'track' (so that it will not roll). Using the meter stick with the calipers, adjust the moveable caliper until the tennis ball fits snugly between the metal caliper 'jaws' (but *without* being deformed). Record the positions of the calipers on the metre stick (**in cm**) to measure the diameter D of a single tennis ball:

$D = D_f - D_i =$ _____ $=$ _____.

3. [**2 marks**] Place ten (10) tennis balls snugly side-by-side on the provided 'track'. Using the meter stick with the calipers, record the length L_{10} of all ten tennis balls (**in cm**):

$L_{10} = L_f - L_i =$ _____ $=$ _____.

Use this value to determine the *average diameter* $D_{avg} = L_{10}/10$ of a single tennis ball; show ALL of your work & watch your sig figs. **What benefit(s) are there to taking the average of ten tennis balls over simply using our measurement from a single tennis ball?**

4. [**1 mark**] Use your calculated average tennis ball diameter D_{avg} and Equation 2 to calculate the (average) volume of a tennis ball, $V_{avg\,ball}$. Show ALL work.

5. [**1 mark**] Using the meter stick with the calipers, record the height H_{bucket} and the diameters of the top AND bottom of the white cylindrical bucket (**in cm**):

$H_{bucket} = H_f - H_i =$_____ $=$ _____.

$D_{top} = D_f - D_i =$_____ $=$ _____.

$D_{bot} = D_f - D_i =$_____ $=$ _____.

Use D_{top} and D_{bot} to calculate the average diameter D_{avg} of the bucket. Show ALL work.

6. [**1 mark**] Use your values for the height of the bucket H_{bucket}, the average bucket diameter D_{avg}, and Equation 3 to calculate the volume of the white bucket, V_{bucket}. Show ALL work.

7. [*2 marks*] How many tennis balls will fit in the white bucket? To calculate this, divide the volume of the bucket V_{bucket} by the (average) volume of a tennis ball, $V_{avg\,ball}$; show ALL work.

# of tennis balls in bucket =_____.

Figure 2: spheres packed randomly into a cylinder

The above *overestimates* the number of tennis balls that fit in the bucket because it assumes ALL of the volume in the bucket is *completely* filled with tennis balls! However, it is apparent by looking in the bucket when it is 'filled' that *there is a considerable amount of empty space between the tennis balls* as they do not 'align' precisely and thus create gaps in three dimensions (see Figure 2 above). When thrown randomly into a container (*'random loose packing'*), tennis balls only occupy about 55% of the actual volume, so revise your original estimate to account for the 'empty' space:

# of tennis balls in bucket (revised) = _____$\times 0.55 = $_____.

Use *percent deviation* to *compare* your revised estimate with the actual value (94). *Comment.*

8. Tidy the apparatus.