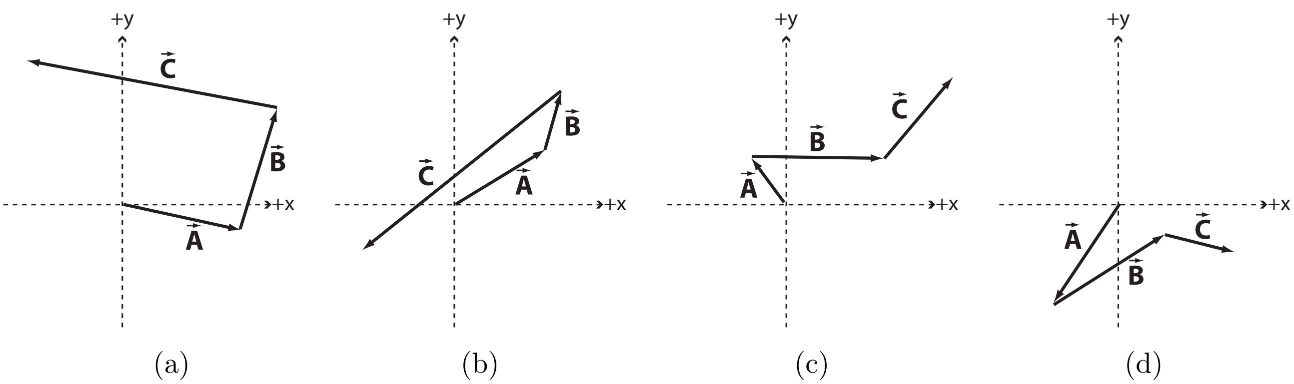
Laboratory 2 Pre-Lab (value: *2 marks*)

Submit to your lab instructor ***by 4pm the day BEFORE*** your lab period.

1. What condition(s) must be met by an object in *static equilibrium*?

2. Draw $\vec{\mathbf{D}}$ that 'closes' the polygons shown & measure Θ, its direction CCW from the +x-axis:



3. Calculate the x and y components of a force of 11.6 N at 171°. Show all work & watch signs!

4. Put F_x = -11.5 N, F_y = 1.81 N into *magnitude & direction* form, with the angle CCW to +x-axis.

Laboratory 2:
Applications of Static Equilibrium to Models

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Arm model, bench clamp, stand, two 40 cm rods, cross clamp, short rod, set of hooked masses, set of slotted masses, set of kilogram masses (1.000kg, 0.500kg, 2.000kg), large mass hanger, metre stick and support clamp with black pointer, short wooden metre stick with calipers. Students supply their own 30 cm plastic ruler, protractor.

OBJECTIVE

1. To apply the principles of static equilibrium to a model of a human arm.
2. To determine the reaction force at the elbow joint of a loaded arm.

THEORY

Models are frequently used as an aid in studying the properties of physical systems. Since real systems are typically quite complex, all models involve some degree of simplification and approximation. However, even a relatively simple model can be useful in providing insight into the characteristics of a real system. In this experiment, wooden pieces joined with bearings are used to model the behaviour of a human arm under load. The biceps muscle is modeled by a tension spring. The magnitude and direction of the force on the elbow joint are found by applying the conditions for static equilibrium.

Static equilibrium in two dimensions

According to Newton's second law, the two conditions for static equilibrium require that

- the vector sum of the external forces acting on the body is zero, and
- the vector sum of the external torques acting on the body is zero.

In this lab we study the first condition for equilibrium; in the next lab we examine the second.

Graphical addition of vectors

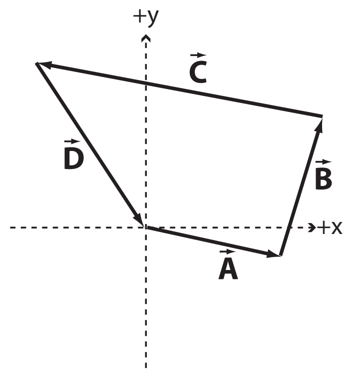


Figure 1: Vector polygon

In the polygon method of vector addition, scaled vectors (whose measured lengths correspond to the magnitudes of the forces that they represent, e.g. 1 cm = 3 N) are drawn such that each vector starts at the end point of the previous one and points in the same direction as the force they represent. In Figure 1, suppose that the vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ are all of the forces acting on an object in static equilibrium. The sum of these forces *must* be zero, as shown in the figure (i.e. adding these vectors in any order produces a *closed* polygon). Any number of vectors can be added head-to-tail in this way.

Analytical addition of vectors

Components of a vector

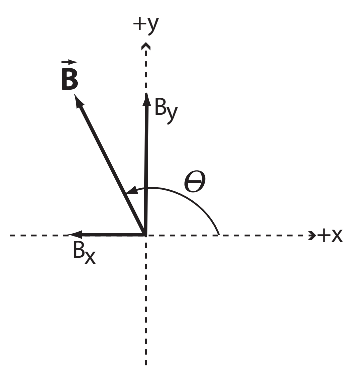


Figure 2: Vector components

The vector $\vec{\mathbf{B}}$ and a set of orthogonal axes (x & y) are shown in Figure 2. The components, B_x and B_y, are the projections of $\vec{\mathbf{B}}$ on the two axes. The components are given by

$$\begin{aligned} B_x &= B\cos\theta \\ B_y &= B\sin\theta \end{aligned}$$

By convention, the angle θ is measured **counterclockwise from the positive x-axis** to the arrow end of the vector, and has a value between 0 and 360 degrees. If this convention is followed, the algebraic sign of the trigonometric function will automatically indicate whether a component is positive or negative (i.e. you do not need to manually 'add' the sign).

Resultant, magnitude and direction

If the components of a vector are known, the *magnitude and direction* of the vector are found by

$$|\vec{\mathbf{B}}| = \sqrt{B_x^2 + B_y^2} \tag{1}$$

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) \tag{2}$$

Note that your calculator will always yield a value of θ that places the vector in Quadrant I or IV; you must, on the basis of the *signs* of B_x and B_y determine which Quadrant the vector falls in, and *possibly* add 180° or 360° to θ as appropriate.

Addition of components

The x and y components of the sum of the addition of several vectors are, respectively, the sum of the x components of the individual vectors and the sum of the y components of the individual vectors. For example, if $\sum \vec{\mathbf{F}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$, then

$$\sum F_x = A_x + B_x + C_x + D_x$$
$$\sum F_y = A_y + B_y + C_y + D_y$$

where the x and y components for each vector are found as shown previously. From these sums the magnitude and direction of the resultant or net force can then be found. If an object is in *equilibrium*, then $\sum F_x = \sum F_y = 0$.

Arm model

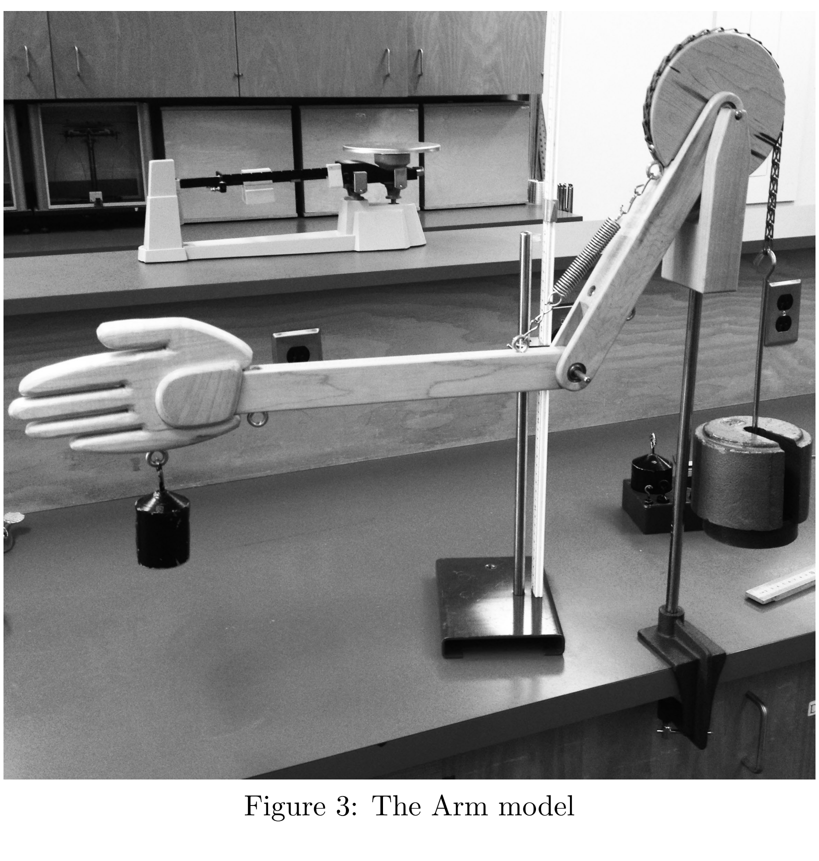


Figure 3: The Arm model

NAME:
DATE: PARTNER:

Laboratory 2: Applications of Static Equilibrium to Models

Setup and Measurements

1. Set up the arm as directed (see Figure 3), **MAKING SURE the hand 'points' to the LEFT**.

2. Hang a hooked mass of 500. grams from the hand ($m_{load\,on\,hand}$) THEN place a counterweight of approx 5.000 kg on the hanger hooked to the vertical shoulder chain; the hanger itself has a mass of 1.000 kg. The lower arm should be roughly horizontal & the upper arm should NOT touch the shoulder support; adjust the apparatus by adding more mass to the hand and/or CW as required. Displace the hand slightly from equilibrium; the system should oscillate freely and return to rest.

3. [***2 marks***] Draw a FULLY LABELLED view of the apparatus in the space below. Record the mass added to the hand (eg. $m_{load\,on\,hand} = 500.$ g) and counterweight, L & Δy, etc.

4. [***1 mark***] Attach the calipers on the wooden metre stick, spaced so that they just fit over the **OUTERMOST loops of the STRETCHED biceps spring**. Lock them in place and **LEAVE the calipers set at this length for the REMAINDER OF THE LAB.** Record the positions of the calipers on the metre stick (**in cm**) to measure the length, L, of the stretched spring:

$L = L_f - L_i =$______________________________ = ____________.

5. [***1 mark***] Mount the plastic metre stick vertically on the support stand and use the black pointer to measure (**in cm**) the VERTICAL positions (y-coordinates) of the **OUTERMOST** loops of the stretched bicep spring (i.e. the same points used to measure L above):

$\Delta y = y_f - y_i =$______________________________ = ____________.

6. Remove the large mass from the hanger, THEN the mass on the hand; dismantle the arm model.

7. [***1 mark***] Place lower arm & hand on the balance. Record their combined mass. Convert to kg.

$m_{arm+hand}$ =______________________________ = _______________.

8. [***1 mark***] Mount the cross bar FIRMLY at the top of the support rod and hang the bicep spring from it. Attach the 1.000 kg mass hanger DIRECTLY to the lower end of the spring and add mass (start with kilogram masses and then use smaller masses) to the hanger until the spring is stretched to the SAME length L as when it was on the arm model (use the UNALTERED spacing of the calipers on the wooden ruler for comparison/verification). Record the total mass required:

$m_{stretch}$ =______________________ + 1.000 kg (hanger) = _______________.

9. Tidy the apparatus.

Analysis and Results

**** Assume the gravitational acceleration to be $g = 9.81$ m/s^2 and to have 3 sig figs. ****

1. [***1 mark***] Calculate $F_g = W_{total} = mg = (m_{arm+hand} + m_{load\,on\,hand})g$, the total gravitational force (weight) of the lower arm, hand and the mass added to hand. Show ALL work.

2. [***1 mark***] Calculate the magnitude of the biceps muscle force, $F_b = mg$, using the mass $m_{stretch}$ required to stretch the spring. Watch your units. Show ALL work.

3. [***1 mark***] Using the previously measured length L of the stretched spring & the vertical difference Δy of the ends of the biceps spring, calculate the angle that the spring makes with the horizontal: $\theta_b = \sin^{-1}(\Delta y/L)$. This angle is also the *direction of the bicep force*, $\vec{\mathbf{F}}_b$. Show ALL work.

4. [***6 marks***] Apply the graphical (vector polygon) method to determine the (unknown) elbow force. **ON THE SEPARATELY PROVIDED GRAPH PAPER, draw a large scale vector polygon of the known forces (bicep, gravitational) acting on the lower arm**. Once all known forces are drawn 'tip-to-tail', draw the vector which must be added to the others in order to 'close' the polygon (i.e. result in a net force of zero). This vector represents the (unknown) force provided by the elbow joint, $\vec{\mathbf{F}}_e$, which balances the other (known) forces.

Use a sharp pencil, ruler and protractor when drawing vectors. ***Label*** **all forces, angles, the x & y coordinate directions, and the scale used**, eg. 1 cm = 2.5 N or 1 cm = 3 N. Choose a scale that maximizes vector size but still fits on the page. ***Neatly*** **show all scaling calculations ON THE GRAPH PAPER**. Measure the length of the elbow force on your diagram & use your scale factor to convert this length to the **magnitude of the elbow force**. Measure the angle that the elbow force makes *counterclockwise from the positive x-axis* to determine the **direction of the elbow force**. Label both the magnitude & direction/angle of the elbow force on your diagram.

5. [***3 marks***] Draw a **FREEBODY DIAGRAM** for the lower arm & hand IN THE SPACE BELOW. The freebody diagram MUST have the same orientation as your arm model setup/drawing from earlier; DO NOT flip left-to-right. Draw and label the 3 forces ($\vec{\mathbf{F}}_g$, $\vec{\mathbf{F}}_b$, $\vec{\mathbf{F}}_e$) acting on the system, including the angles at which they act. Angles should be referenced counterclockwise from the positive x-axis. Use SYMBOLS ONLY (e.g. $\vec{\mathbf{F}}_b$, θ_b) in this diagram, NOT numbers. **Magnitudes & directions for the forces are meant to be approximate**. **In words, JUSTIFY your reasoning** in choosing the (approximate) direction & magnitude of the unknown force $\vec{\mathbf{F}}_e$.

6. The following questions use the analytical (vector components) method to determine (the unknown) $\vec{\mathbf{F}}_e$. Please note that this method is **INDEPENDENT** of your vector polygon, so DO NOT use ANY values from the polygon drawing in the steps below. At the end of the lab you will COMPARE the results of the two approaches used to find $\vec{\mathbf{F}}_e$ and see how well they agree.

7. [***2 marks***] Referring to your freebody diagram & USING ONLY SYMBOLS, write the equilibrium equations for the sum of the forces in the x-direction and for the sum of the forces in the y-direction. Recall that for an object in *equilibrium*, $\sum F_x = \sum F_y = 0$. SOLVE the equations *algebraically* for the (unknown) components of the elbow force, F_{ex} and F_{ey}, eg. $F_{ex} = \ldots$ and $F_{ey} = \ldots$.

DO NOT INSERT NUMBERS! Watch signs & show ALL steps. For example, in the x-direction:

$$\begin{aligned} \sum F_x &= F_{gx} + F_{bx} + F_{ex} \;=\; 0 \\ 0 &= 0 + F_b \cos\theta_b + F_{ex} \\ F_{ex} &= -F_b \cos\theta_b \end{aligned}$$

Using SYMBOLS ONLY, derive the corresponding equation for F_{ey} in the y-direction:

8. [***3 marks***] Substitute the numerical values F_g, F_b & θ_b **calculated** earlier in the lab (NOT from the vector polygon) into your equations (above) and calculate F_{ex} & F_{ey}. Show ALL work.

9. [***2 marks***] Calculate the magnitude & direction of the elbow force using F_{ex} & F_{ey} (above) and Equations 1 & 2 from the Theory. Reference the angle CCW from the +x-axis. Show ALL work.

10. [***3 marks***] Compare the magnitudes ($|\vec{\mathbf{F}}_e|$) found using the graphical and analytical methods with *% difference* (see previous lab). Repeat for the directions (Θ_e). Comment on your comparisons & their agreement. Which method *would you expect* to be more accurate, typically? Explain.