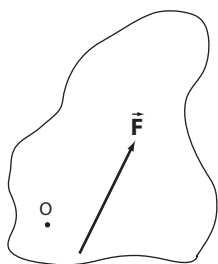# Laboratory 3 Pre-Lab (value: *2 marks*)
Submit to your lab instructor *by 4pm the day BEFORE* your lab period.
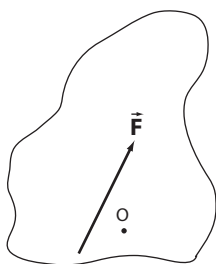
1. Describe the lab procedure to determine the *centre of gravity* (CG) **of the metal plate**.

2. Where will the *net torque* be greatest for an object in *static equilibrium*? Explain briefly.

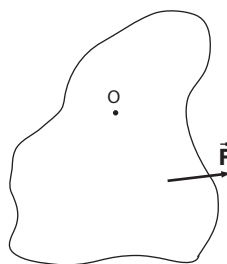3. Indicate the *sense* or *sign* ($\pm$) of the torque about the point O due to $\vec{F}$ for each case below:
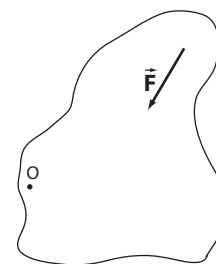

(a)   (b)   (c)   (d)
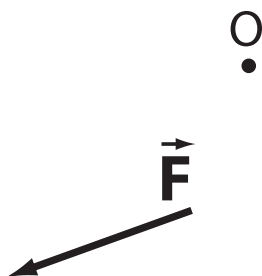
4. Measure the *lever arm* $r_\perp$ (from point O to *the line of action* of the applied force $\vec{F}$) with a ruler & right triangle. Given $|\vec{F}| = 2.0$ N, calculate the (vector) torque $\vec{\tau}$ about O. Show ALL work!

Laboratory 3:
Torques and Non-Concurrent Forces

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Vertical force board, 2 C-clamps, set of 3 strings, triple-beam balance, 2 sets of hooked masses, irregular metal plate, hook, plain paper, masking tape, Tee square. Students supply their own 30 cm plastic ruler, triangle.

OBJECTIVE

1. To locate the centre of gravity of a plane body.

2. To confirm the conditions for static equilibrium of forces in two dimensions.

THEORY

A set of forces rarely acts through a single point on a body (i.e. concurrent forces). Instead, each force typically acts at a different point on the body and, in doing so, tends to cause the body to rotate. This twisting force acting about some chosen axis is called a torque. Familiar examples of torques are the use of a wrench or the turning the steering wheel of a car. Many body muscle groups exert torques on the joints in order to rotate some part of the body, e.g. the neck and head.

In this lab, the centre of gravity of a plane body is determined. A static system of non-concurrent forces is applied to the body such that it is in equilibrium. The lines of action of the forces are traced onto a sheet of paper and the resulting torques produced around an (arbitrary) chosen point are calculated. The torques are summed to verify that the net torque on the body is zero.

Centre of gravity of a plane body

The centre of gravity of a body is defined as the point at which its weight appears to act.

If a plane body is suspended freely, its centre of gravity lies directly beneath the point of suspension; that is, a line drawn vertically downward from the point of suspension passes through the centre of gravity. If another point of suspension is selected and the process is repeated, the intersection of the two lines marks the location of the centre of gravity.

Static equilibrium in two dimensions

According to Newton's second law, the two conditions for static equilibrium require that

- the vector sum of the external forces acting on the body is zero, and

- the vector sum of the external torques acting on the body is zero.

In this lab, we will focus on the second condition for equilibrium.

Torque

If a force $\vec{F}$ is applied to a body, the torque produced about some arbitrary point O located a perpendicular distance $r_\perp$ from the *line of action* along which the force acts is

$$\tau \;=\; F \cdot r_\perp$$

where $r_\perp$ is also referred to as the *lever* or *moment arm*. The second condition for equilibrium requires that the **sum of the torques about any point is zero**. Since the forces are assumed to lie in a plane, the individual torques will tend to rotate the body in either a counter-clockwise direction or a clockwise direction about the chosen point. **By convention, counter-clockwise (CCW) torques are considered to be positive while clockwise (CW) torques are negative**.
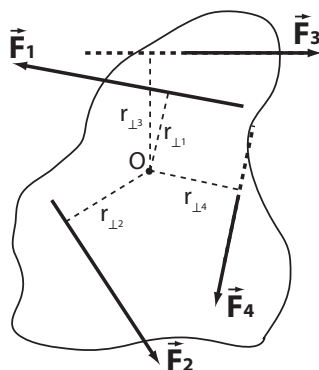

Figure 1: Torques about the point 'O'

The forces acting on a planar body are shown in Figure 1. An arbitrary point 'O' about which to sum the torques has been chosen and the *lever arms* (perpendicular distances from the lines of action of each of the forces to the chosen point) have been drawn. In this example, $\vec{F_1}$ and $\vec{F_2}$ will tend to rotate the body in a counter-clockwise (positive) direction about point O while $\vec{F_3}$ and $\vec{F_4}$ will tend to rotate the body in a clockwise (negative) direction. The second condition for equilibrium will be satisfied if the algebraic sum of the four torques is zero. For the above example

$$\sum \tau \;=\; \tau_1 + \tau_2 + \tau_3 + \tau_4 \;=\; +(F_1 \cdot r_{\perp 1}) + (F_2 \cdot r_{\perp 2}) - (F_3 \cdot r_{\perp 3}) - (F_4 \cdot r_{\perp 4}) \;=\; 0$$

where $r_{\perp i}$ is the perpendicular distance from the point O to the line of action of the ith force and we have used the standard convention regarding the direction of torques to determine the signs.

Laboratory 3: Torques and Non-Concurrent Forces

Centre of gravity, force board setup

1. Cut a piece of paper to fit and tape it onto the central portion of the metal plate.

2. **Determine the centre of gravity of the metal plate** as follows. Suspend the plate freely from a stable location using a paperclip through any of the holes in the plate. Hang a short string from the paperclip and add a small mass to the lower end of the string to form a plumb-bob. Carefully mark the position of the vertical string on the paper at two widely-spaced points. Suspend the plate at a different orientation and repeat the process. Repeat for a third orientation of the plate.

3. Connect each pair of points *using a ruler*. If the resulting lines DO NOT cross exactly at (or VERY close to) a single point, add additional orientations/lines until three of them cross at (VERY close to) a single point; this point marks the centre of gravity of the plate.

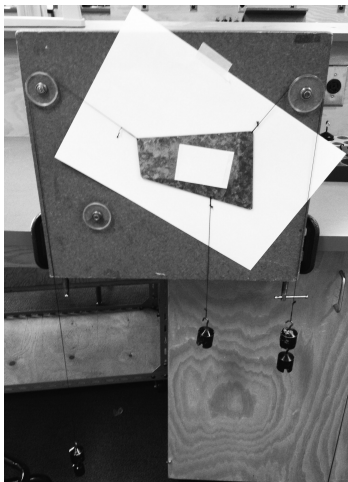4. **LEAVE the paper on the plate** for the remainder of the lab.

Figure 2: Vertical force board

5. Set up the force board (Figure 2) as directed by your instructor, clamping it securely to the bench. Using the strings and the masses supplied, **set up a system of three forces so that the plate is suspended in equilibrium near the centre of the force board**. Attach the two *longer* strings OVER the top pulleys. Attach the *shorter* string to the bottom of the plate so it HANGS VERTICALLY (does NOT pass over a pulley) and is NOT close to being immediately below the center of gravity of the plate. Thread the strings through the holes in the plate and back upon themselves to secure them - DO NOT TIE KNOTS. Start with 200.g on the pulley strings & 100.g on the vertical string. Make certain all masses DO NOT touch the force board (i.e. hang freely) and that EACH string has AT LEAST 100.g. Add mass as needed to center the plate on the force board. AVOID USING MANY SMALL (<50.g) MASSES.

Displace the plate slightly from equilibrium; the system should oscillate freely and (eventually) return to rest. Tap (vibrate) the force board in order to minimize static friction in the pulleys.

6. Slide a (legal sized or larger) sheet of paper between the plate and the force board and orient it to MAXIMIZE the length of each string located over the paper. Adjust the masses if necessary. Tape the paper in place; the paper's edges need NOT align with the force board (i.e. the paper can be at an angle on the board). **Partners set up the apparatus together but EACH person makes their OWN drawing and does their OWN analysis.**

7. WITHOUT shifting the plate's position, press it against the paper/board and hold it in place. TRACE the outline of the plate onto the paper. Carefully mark (NOT TRACE) the position of each string with small dots at **two widely-spaced points**; use a sharp pencil and be as accurate possible. **Label each pair of marks with the value of the mass (in grams) on the string and draw a short arrow to indicate the direction of each force. Draw an arrow labelled DOWN to indicate the downward (vertical) direction. Put your NAME on the paper**.

8. Remove the sheet of paper from the force board. Complete the outline of the plate if necessary. Connect the pairs of points marking the position of the strings *with a ruler* to yield the line of action for each force; **EXTEND these lines of action to FULLY cross the paper**. **Label the forces** using the names $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$.

9. REPEAT the tracing process above for EACH lab partner.

10. Tidy the apparatus. **DO NOT REMOVE** the centre of gravity paper from the metal plate.

Analysis and Results

1. A data table is given on the last page; do ALL calculations in the space **BELOW THE TABLE**.

2. Place the plate into the position traced on your paper and extend the centre-of-gravity lines with a ruler from the plate onto your sheet. Remove the plate and complete the line segments. **Label the point of intersection as CG**. (**NOTE**: DO NOT extend these lines across the entire paper; **draw them as short as possible to form a '*' near the centre** to reduce visual 'clutter').

3. Measure the **plate's mass** (including paper but excluding strings) & record it and the **letter stamped on the plate** in the spaces provided ABOVE the data table at the end of the lab.

4. Remove the piece of paper from the plate. Include it in one partner's write-up.

5. **Enter the masses** (converted from grams to kilograms) of the plate and corresponding to each force in the space provided in the table. **Assume all hooked masses are good to the nearest gram**, eg. 50.g = 0.050 kg; **the plate's mass should be better determined** and good to the nearest 0.1 g = 0.0001 kg. Show a sample calculation of a mass conversion below the table.

6. **Calculate the weight** (in newtons, N) of each of the masses & the plate using $F_g = mg$ and record the results in the table. Underline the *least significant digit* & **keep ONE extra (non-significant) digit in your table entries**. Show ONE full sample calculation below the table.

** Assume the gravitational acceleration to be $g = 9.81$ m/s^2 and to have 3 sig figs. **

7. Establish the line of action of the weight of the plate (i.e. downward force of gravity on the plate).

Place your drawing on the lab bench and set the tee square on top of it. Hold the black head of the tee square against the edge of the bench. Position the drawing under the tee square so that the line of action of the vertically hanging weight (i.e. the force corresponding to the string not running over a pulley) is parallel to one edge of the tee square. Tape the paper to the bench in this position.

Slide the tee square along the edge of the bench until the edge of the tee square aligns with the centre of gravity of the plate (marked CG on your drawing). **Draw a line through the centre of gravity (and parallel to the line denoting the vertically hanging mass) across the entire object. Label it as F_g and mark the mass of the plate (in grams) next to it**.

Hint: You will end up with a LOT of lines on your drawing, so use dashed lines, coloured pencils, highlighters, etc. to differentiate lines denoting forces, perpendicular distances, lines of action, etc.

8. Choose & label a pivot point 'O' that is INSIDE the outline of your plate about which to sum the torques. Any point *should* work; however, to reduce error **DO NOT CHOOSE the centre of gravity (CG) or a point which lies within 1 cm of the line of action of any of the forces**.

9. Using a sharp pencil, a ruler, and a triangle, **draw perpendiculars** from the (extended) lines of action of the forces to the point 'O'. Label the perpendicular distances from each of the force lines to the point 'O' as $r_{\perp i}$ (as shown in Figure 1). **PROPERLY measure these distances (in cm), recording your $r_{\perp i}$ values (in m) in the table**. Show a sample calculation below the table.

10. **Determine the sense** (CCW = '+', CW = '-') of each of the torques about your point 'O'. To ascertain this press down on the point 'O' with a finger of one hand and use the palm of your other hand to push on the paper *in the direction of a force*, along its line of action; this should rotate the paper either CW ('-') or CCW ('+') about 'O'. **Record the sign in the 'sense' column of the table** (no 'work' needs to be shown for this entry).

11. **Calculate the individual torques** (in Nm) & **record them in the table**. Underline the *least significant digit* & keep ONE extra (non-significant) digit in your table entries. Show a full sample calculation below the table.

12. **Sum the torques** (including the extra non-sig digit) at the bottom of the torque column in the table, showing a sample calculation below the table. **Round this sum of *unrounded* torques** to the *correct* number of sig figs (i.e. one fewer) and **record this properly rounded sum in the space provided in the table** (i.e sum of torques with correct sig figs).

The sum of the torques should be (very close to) zero; in practice, this means the sum is smaller than the magnitude of any of the individual torques. If your sum is NOT (close to) zero, as a first step **CHECK the signs of your torques and the measured values of your $r_\perp$'s**. IF these seem correct and the sum of the torques remains far from zero then speak with your lab instructor.

13. [*4 marks*] Staple your drawing to your lab; make sure it can be EASILY accessed for marking.

14. [*1 mark*] m_{plate} =_____. Letter stamped on plate: _____.

15. [*5 marks*] Complete the table below with data collected from your drawing. **A FULL set of sample calculations is REQUIRED for ONE complete row of your table AND for the sum of the torques calculation**. Perform your calculations in the space BELOW the table (and on the back of the last page if extra space is required). Show ALL work FULLY.

force	mass (kg)	weight (N)	$r_{\perp i}$ (m)	sense ($\pm$)	torque (Nm)
1					
2					
3					
Plate					
$\sum \tau$ (with extra non-sig digit) =					
$\sum \tau$ (rounded to correct sig figs) =					

16. [*2 marks*] **Using your non-rounded (extra digit) torques** calculate the sum of the positive (CCW) torques ($\sum \tau_{CCW}$) and the *absolute value* of the sum of the negative (CW) torques ($|\sum \tau_{CW}|$). *If there was no experimental error these sums should be identical to each other, resulting in a net torque of exactly zero.* **Determine how close you come to this by using your calculated sums (above), your value for the (unrounded) sum of the torques (Table) and the (modified) percent difference equation provided below**. Show ALL work/steps, rounding the percent difference to 1 or 2 digits as we did in previous labs.

$$\text{modified \% difference} \quad = \quad \frac{|\sum \tau|}{\left(\dfrac{\sum \tau_{CCW} \; + \; |\sum \tau_{CW}|}{2}\right)} \times 100$$

17. [*1 mark*] The hooked masses may be 1% different than their stamped values and so a calculated percent difference (above) of less than this is within the uncertainty in the masses. What other **significant** sources of error might there be for this lab (*be specific!*)?