Introduction to Measurement & Calculation

Significant Figures

Significant figures indicate the *reliably known digits* in an experimental value; they are typically *determined by the precision of the instrument(s) used to make the measurement(s).* When performing calculations using measured values, keep in mind that not all digits in the answer displayed by your calculator are necessarily significant (*see* **Calculation & Significant Figures**).

In the lab we observe the following (standard) conventions regarding significant figures:

1. All non-zero digits ARE significant
2. Zeroes *between* non-zero digits ARE significant
3. *Leading zeroes* are NOT significant
4. *Trailing zeroes* ARE significant *IF* there is a decimal point
5. All digits ARE significant when a number is expressed in *scientific notation*

(Example) *The numbers 19.7, 80, 0.0031 and 0.2050 have 3, 1, 2 and 4 significant figures respectively. As written, 15,600 would have 3 sig figs; if written as 15,600. (with an explicit decimal point) it would be considered to have 5 sig figs. One could avoid potential confusion by using scientific notation, i.e. writing* 1.56×10^4 *for 3 sig figs or* 1.5600×10^4 *for 5 sig figs.*

Precision and Accuracy

Precision and **accuracy** are terms which are often (*incorrectly!*) used interchangeably.

Precision refers to how well a value is known. There is no such thing as an exact measurement; there is *always* some error or uncertainty associated with it. The uncertainty in a measurement may be very small, but it is never zero. The uncertainty describes the range of values within which you are reasonably sure the measurement lies. The smaller the uncertainty, the more precisely the value is known. Precision may be indicated by the number of decimal places in a value.

(Example) *The acceleration due to gravity is found by various experimental methods to be 9.8, 9.81 and 9.805* m/s^2*. The third value, 9.805* m/s^2*, is the most precise because it is determined to within one thousandth of a* m/s^2 *(3 decimal places). The first value, 9.8* m/s^2*, is the least precise because it is only determined to one tenth of a* m/s^2 *(1 decimal place).*

Accuracy refers to how close a result is to the actual or true value.

(Example) *9.7* m/s^2 *is a more accurate value of the acceleration due to gravity than 9.6* m/s^2 *is, since the expected value is 9.8* m/s^2 *(to two significant figures). The values are equally precise, however, since they are both determined to a tenth of a* m/s^2 *(1 decimal place).*

Recording of Experimental Values

In the physics lab experimental values will be recorded to the precision of the measuring instrument, which is taken to be the smallest division indicated on the instrument. This must however be tempered with reasonable judgment as cases may arise where the precision of the measurement is much less than the (theoretical) precision of the measuring instrument.

It is good experimental practice to record all measurements *as read directly from an instrument or measuring device*, without any unit conversion or calculation. For example, if a scale used to measure mass reads in grams, then record any measurements made with that instrument in grams. If necessary, convert to kilograms or other units only *after* the measurement is made and recorded in grams.

Metre Stick (metric scale or metric ruler)

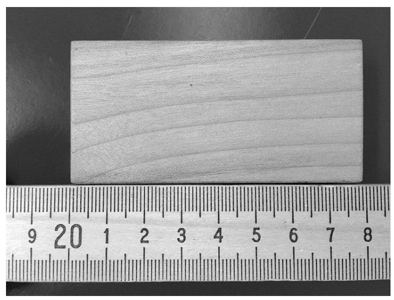


Figure A: measurement of a rectangular block

The familiar metre stick is usually divided into centimetres and tenths of centimetres (millimetres), with the centimetres being numbered - for example, see Figure A. The smallest divisions are tenths of a cm (1 mm) and therefore the precision of the metre stick is 1 mm or 0.1 cm. If the ruler is transparent and the graduations are on the underside, it may be laid flat on the object. If the ruler is not transparent, or if the graduations are on the upper surface, the ruler should be held on edge so that the graduations are in contact with the object. This eliminates errors due to parallax.

A linear distance measured with a ruler consists of two readings, which are the positions of both ends of the object. Typically, the “zero” end of a metre stick is in poor condition and is *not* used as one of the positions; otherwise, any other convenient line on the metre stick may be used. In one step, a complete length measurement would be recorded as:

Length = (position of one end of block) - (position of other end of block) = (length of block)

(Example) *The measurement of the length of the block in Figure A would be recorded as:*

$$Length \;=\; 27.9\ cm - 20.0\ cm \;=\; 7.9\ cm$$

Lengths may be measured *directly* if a (plastic) ruler has a well defined “zero” (i.e. using 0.0 cm).

Triple Beam Balance

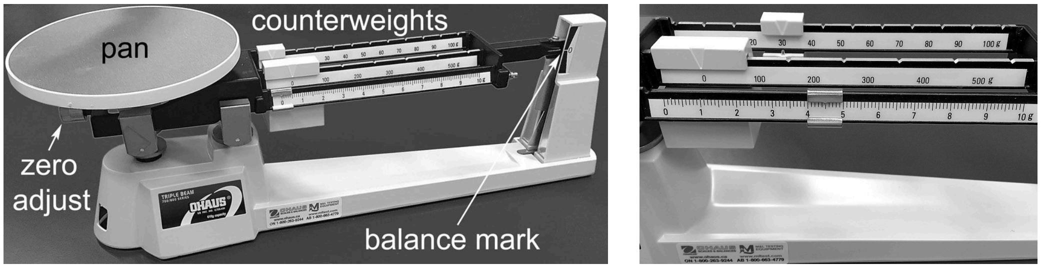


Figure B: Triple beam balance (left); closeup of counterweights when measuring (right).

The triple-beam balance derives its name from the three beams which carry the counterweights that balance the weight on the pan. The balance determines the mass m of an object by exploiting the linear relationship $W = mg$ between an object's weight W & the gravitational acceleration g; this allows the balance to be graduated directly in mass units (e.g. readings yield mass in grams).

The object whose mass is to be determined is placed on the pan and a balance condition is achieved by moving, *in order*, the counterweights on the hundreds beam, the tens beam and the units beam. A static balance condition is indicated by the pointer on the beam being lined up with the central mark on the scale on the end frame. The hundreds and tens beams have notches in which the counterweights *must* sit in order for them to be in the correct positions. The units counterweight may be placed at any position along its beam, the position being shown by the pointer in the centre of the counterweight. The smallest division on the units beam is 0.1 g and therefore the precision of the triple-beam balance is ± 0.1 g, *i.e. to the nearest tenth of a gram.*

The balance is subject to a zero error, i.e. when all the counterweights are set at 0, the pointer may not be lined up with the central mark on the frame. The zero point is adjusted by turning the adjustment knob beneath the pan. The zero reading *must* be properly set BEFORE using the balance.

(Example) *Assuming proper zeroing, the balance reading in Figure B is* $30.0 + 0.0 + 4.5 = 34.5$ *g.*

Calculation & Significant Figures

1). When **adding or subtracting** numbers, the precision of the sum or difference is the same as that of the *least precise number* in the set.

(Example) $4.513 + 3.9 = 8.413$ *is rounded to 8.4 because 3.9 is only good to one decimal place.*

(Example) $110 + 39.5 = 149.5 = 150$ *since 110 (*$=1.1 \times 10^2$*) is only good to the 'tens' place.*

2). When **multiplying or dividing** numbers, the number of significant figures in the product or quotient is the same as that of the number in the set which has the *fewest significant figures.* **Note:** multiplication or division by a constant does *not* affect the determination of significant figures.

(Example) $\dfrac{4.513 \times 7.8}{2.001} = 17.5919$ *is rounded to 18 because 7.8 has only two significant figures.*

3). When dealing with **mixed calculations** (e.g. combinations of addition, subtraction, multiplication and division one calculation) it is *necessary* to perform the calculation in steps, obeying *mathematical order of operations*, in order to determine the proper significant figures in the final answer. ***You CANNOT do such calculations in a single step on your calculator and properly account for significant figures.*** To **minimize round-off error** *it is common practice to retain (at least) one EXTRA digit* during steps in a calculation *and mark the position of the least significant digit by underlining it, e.g.* $7.02\underline{3}6$ *or* $591.\underline{4}22$ where the final 6 or 22 to the right of the underlined digit are NOT significant.

(Example) *Calculate the volume of a rectangular solid if the length, width and height are measured to be 16.0 cm, 7.95 cm and 3.9 cm, respectively.*

$$V \;=\; L \times W \times H \;=\; (16.0)(7.95)(3.9) \;=\; 4\underline{9}6.1 \;=\; 5.0 \times 10^2 \; cm^3$$

Since all of the operations are multiplication/division, we can do this calculation in one step, applying the rule for sig figs & multiplication/division. In this case our final answer should have two sig figs, since that is the lowest number of sig figs in the product; note that this results in rounding UP to 500 and that scientific notation is required to clarify that only two digits are significant.

(Example) *Calculate the density of the solid (above) if its mass is measured as 280.9 g.*

$$\rho \;=\; \frac{m}{V} \;=\; \frac{280.9}{4\underline{9}6.1} \;=\; 0.5\underline{6}6 \;=\; 0.57 \; g/cm^3$$

Since all of the operations are multiplication/division, we can do this calculation in one step, applying the rule for sig figs & multiplication/division. In this case we use the ***unrounded*** *value for the volume while noting the least significant digit for the purposes of rounding our final answer.*

(Example) *Calculate the surface area of a rectangular solid if the length, width and height are measured to be 16.0 cm, 7.95 cm and 3.9 cm, respectively.*

$$\begin{aligned}
\text{Area} &= 2LW + 2LH + 2WH \\
&= 2(16.0)(7.95) + 2(16.0)(3.9) + 2(7.95)(3.9) \\
&= 25\underline{4}.4 + 1\underline{2}4.8 + 6\underline{2}.01 \\
&= 4\underline{4}1.21 \\
\text{Area} &= 440 \; cm^2 \; or \; 4.4 \times 10^2 \; cm^2
\end{aligned}$$

Using order of operations, the multiplications are performed FIRST. We apply the rule for sig figs & multiplication/division to EACH of the three terms; as each term contains a number with a minimum of two sig figs, this determines what we can retain for the product. Note that we ignore the impact on sig figs of the constant '2' in each term and keep an extra digit (or two) after the underlined significant digit for each term. NOW the addition is performed and we apply the rule for sig figs & addition/subtraction; in this case, our least precise value in the sum is known only to the 'tens' and so we round our final answer appropriately and add units.

IF the area were used in a later calculation we would utilize the ***unrounded*** *value but underline the least significant digit for the purposes of applying the sig fig rules during that calculation.*

Comparing Values

1). ***Percent difference*** compares two *rounded* experimental measurements:

$$\text{Percent Difference} = \frac{(\textit{higher value} - \textit{lower value})}{\textit{average value}} \times 100\%$$

Percent difference is **always positive** and **is reported with at most 2 significant figures**.

(Example) *Two students measure a rectangular block and calculate its volume. One finds the volume to be 27 cm^3 while the other gets 28 cm^3. The percent difference between their results is*

$$\begin{aligned}\text{Percent Difference} &= \frac{(\textit{high} - \textit{low})}{\textit{average}} \times 100 \\ &= \frac{(28-27)}{27.5} \times 100 = \frac{1}{27.5} \times 100 = 3.636\% = 4\%\end{aligned}$$

Rounded for significant figures, the two values differ by 4%.

2). ***Percent deviation*** compares a *rounded* experimental measurement with its expected value[1]:

$$\text{Percent Deviation} = \frac{(\textit{experimental} - \textit{expected})}{\textit{expected}} \times 100\%$$

Percent deviation may be **positive or negative** (i.e. whether the *rounded* experimental value is greater than or less than the expected value) and **is reported with at most 2 significant figures**.

Percent deviation is sometimes also referred to as ***percent error***.

(Example) *A student calculates the density of aluminum to be 2.5 g/cm^3; the published or expected value is 2.70 g/cm^3. The percent deviation is*

$$\begin{aligned}\text{Percent Deviation} &= \frac{(\textit{experimental} - \textit{expected})}{\textit{expected}} \times 100 \\ &= \frac{(2.5-2.70)}{2.70} \times 100 = \frac{-0.\underline{2}0}{2.70} \times 100 = -7.407\% = -7\%\end{aligned}$$

Rounded for significant figures, the experimental value is 7% *BELOW the expected value.*

[1] The *expected value* may be an accepted value for a physical constant like those given in a handbook of constants (e.g. acceleration due to gravity at Earth's surface) or something calculated theoretically using values determined outside of the lab (e.g. using numbers provided by the manufacturer or accepted values for physical constants).

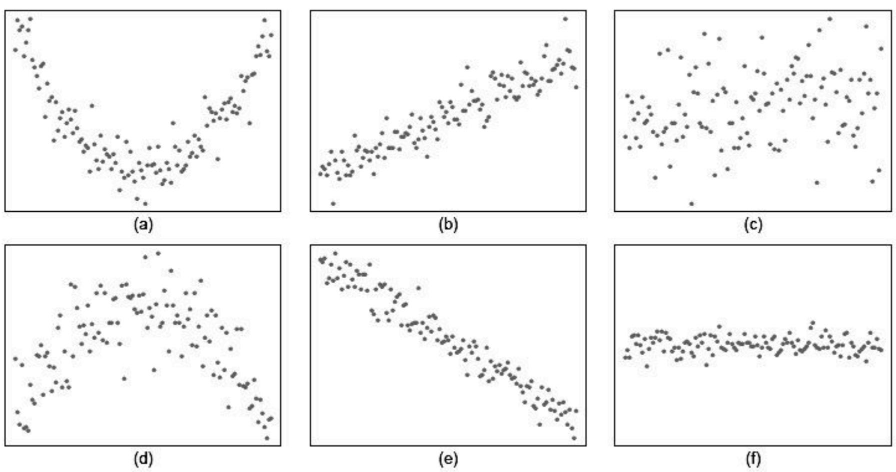
Laboratory 1 Pre-Lab (value: *2 marks*)

Submit to your lab instructor ***by 4pm the day BEFORE*** your lab period.

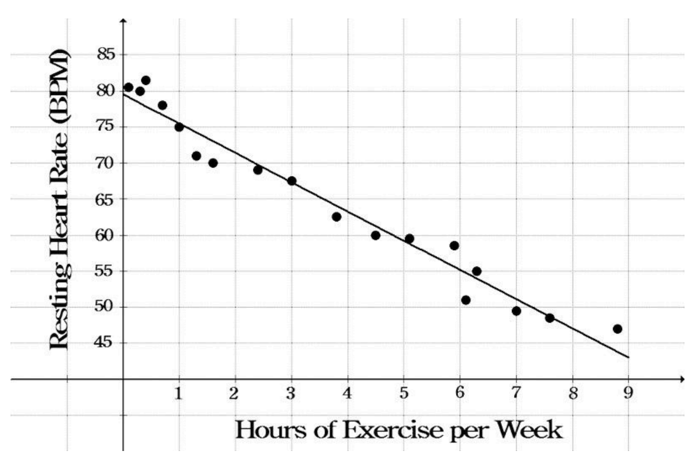
1. How much force is required to stretch a spring 9.0 cm if it requires 22 N to stretch it 3.0 cm?

2. Write down the ***general*** *equation of a straight line* and identify/label each part of it.

3. Circle all *linear* graphs from those shown below and mark them with a '+' , '-' or 'zero' slope.



4. Calculate the slope of the (linear) data shown below. Show *all* work, points chosen, units, etc.



Laboratory 1: Force Constant of a Spring

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Bench stand and support rod, spring, pointer, mass hanger, slotted masses, stop watch, stand with metre-stick clamp, metre stick.

OBJECTIVE

1. To determine the force constant of a spring directly using Hookes law, and indirectly from its period of oscillation.

THEORY

Part A. Hooke's Law applied to a vertical spring

When an ideal spring is stretched by an applied force, the elongation of the spring is proportional to the applied force. This is Hooke's Law for a spring, which may be written as

$$F = kx \tag{1}$$

where k is the force constant of the spring and x measures the amount the spring is stretched. Consider a spring that hangs vertically with a hanger and pointer attached, as shown in Figure 1a.

Applying Hooke's Law, it follows that

$$M_o g = k(y_o - y_u)$$

and

$$(M + M_o)g = k(y - y_u)$$

where M_o is the mass of the hanger & pointer (as well as an extra factor due to the mass of the spring itself) and M is the additional mass added. Rearranging these equations results in

$$y = \frac{g}{k}M + y_o$$

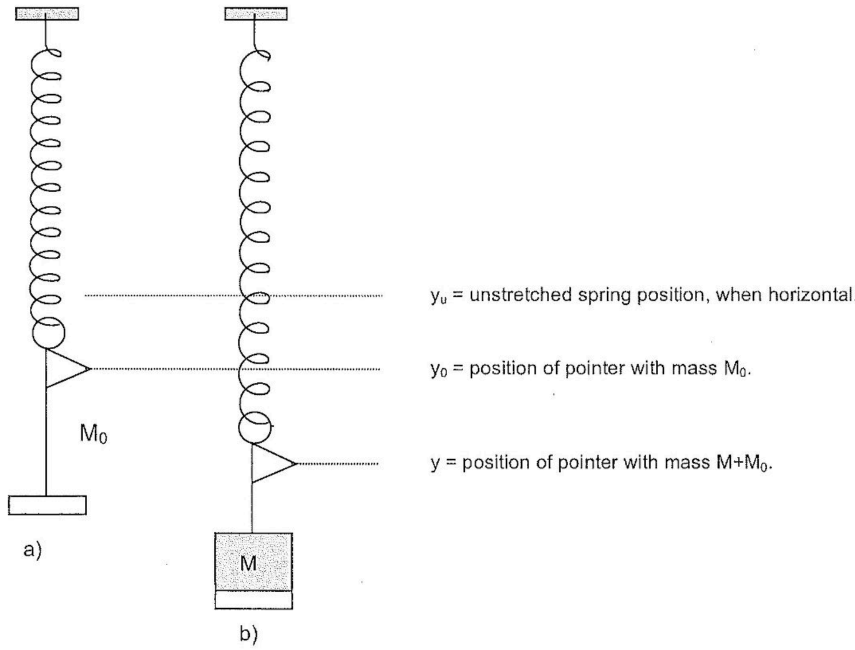


Figure 1: The extension of a vertical spring by the addition of mass.

Comparing the equation above with the *general equation for a straight line*, $y = mx + b$, we see that a graph of y (pointer position) versus M (additional mass added) should be a straight line with a slope equivalent to

$$\text{slope} = \frac{g}{k} \tag{2}$$

Part B. Oscillation of a vertical spring

If the mass $M + M_o$ is pulled down below the equilibrium position shown in Figure 1b and then released, the system will oscillate in simple harmonic motion with a period, T, given by

$$T = 2\pi\sqrt{\frac{M + M_o}{k}}$$

This can be rewritten as

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{k}M_o$$

Comparing the equation above with the *general equation for a straight line*, a graph of T^2 (period squared) versus M (additional mass added) should be a straight line with a slope equivalent to

$$\text{slope} = \frac{4\pi^2}{k} \tag{3}$$

NAME:
DATE: PARTNER:

Laboratory 1: Force Constant of a Spring

DATA COLLECTION

1. Clamp the stand to the bench and suspend the spring from the short horizontal rod. Attach the pointer and the mass hanger to the lower end of the spring ~40 cm above it. Mount the metre stick in the clamp **with the zero end 'up' and the cm scale facing out**.

2. [***4 marks***] Place a 100 g mass (the initial *total added mass*, M) onto the mass hanger and record the *position* y of the pointer, at rest, in the table. Raise the mass hanger slightly ($\sim$ 2 cm with the tip of a pencil) and release it. Make certain the mass is oscillating *vertically* and NOT bouncing side-to-side. **Measure the time** T_{10} for 10 *FULL oscillations* of the system. **Do this a couple of times to make CERTAIN you are getting consistent times AND counting 10 FULL oscillations**. *Hint: Start counting/timing with 0 ('zero') rather than 1 as you only COMPLETE the first oscillation ('one') a full cycle AFTER you begin.* Record a (consistent) 'typical' value for T_{10} in Table 1. Increase M by 50 g and repeat, up to a total added mass of $M = 300$ g. ** NOTE: M is the total ADDED mass & EXCLUDES the mass of the hanger; it is good to ±1g. **

M (kg)	y (cm)	y (m)	T_{10} (s)	T_{avg} (s)	T^2_{avg} (s^2)
0.100					
0.150					
0.200					
0.250					
0.300					

Table 1: vertical spring observational and calculated data.

3. [***2 marks***] **Calculate the average time** for ONE oscillation, T_{avg}, and its square, T^2_{avg}. Show a FULL set of sample calculations for the *entire first row of the table* (i.e. $M = 0.100$ kg).

Part A:

1. [***4 marks***] Plot a graph of pointer position y (m) vs. total mass added M (kg). Fill the graph paper as much as possible and label your graph *FULLY*, i.e. title, axes, units, data points, etc.

2. [***3 marks***] Draw the *line of best fit* for your graph. Pick and clearly mark two (widely separated) and easy-to-read points **ON this line** (they do NOT have to be data points!). *Using these points*, determine the rise (Δy) and run (Δx), using appropriate significant figures and units:

rise (Δy) = ____________________ = _______ *run* (Δx) = ____________________ = _______

Calculate the slope ($\Delta y/\Delta x$) of your line using proper sig figs. Show your work, including units.

3. [***3 marks***] Calculate the force constant k of the spring (in units of N/m) using Equation 2 and your calculated slope (above). Assume $g = 9.81$ m/s^2. Show all work/steps, etc.

Part B:

1. [***4 marks***] Plot a graph of the square of the average period T_{avg}^2 (s^2) vs. total mass added M (kg). Fill the graph paper as much as possible and label your graph *FULLY*, i.e. title, axes, etc.

2. [***3 marks***] Draw the *line of best fit* for your graph. Pick and clearly mark two (widely separated) and easy-to-read points **ON this line** (they do NOT have to be data points!). *Using these points*, determine the rise (Δy) and run (Δx), using appropriate significant figures and units:

rise (Δy) = ____________________ = _______ *run* (Δx) = ____________________ = _______

Calculate the slope ($\Delta y/\Delta x$) of your line using proper sig figs. Show your work, including units.

3. [***3 marks***] Calculate the force constant k of the spring (in units of N/m) using Equation 3 and your calculated slope (above). Show all of your work/steps, etc.

4. [***2 marks***] **Compare** your values of k (using *percent difference*) & **comment** on their agreement.

5. If your graphs are linear AND your k values are consistent then dismantle and tidy the apparatus.