

Laboratory 2: Vibrating Strings

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Mounted ukulele sonometer, moveable bridge, 5 kg counterweight, 2 strings ($\mu_1 = 1.43 \times 10^{-4}$ kg/m, $\mu_2 = 5.72 \times 10^{-4}$ kg/m), mass hanger, 2 sets of slotted masses (2000g total, including 5 and 2 g masses), half-metre stick, tuning meter, pulley with stand setup.

OBJECTIVE

1. To study the properties of standing waves on strings.
2. To find the velocity of a wave traveling along a string.

THEORY

When a string is set into vibration by plucking it standing waves can be generated. Examples of standing waves for a stretched string of length L are shown in Figure 1. The lowest possible frequency (the **fundamental frequency**) is produced when the string vibrates in one segment, as in Figure 1a. The string in Figure 1b is vibrating in its **second harmonic** mode (twice the fundamental frequency). Figure 1c shows the **third harmonic** (three times the fundamental frequency).

The wavelength of the generated waves is equal to twice the nodal separation (positions of zero amplitude); hence, the wavelength corresponding to the n^{th} harmonic is given by

$$\lambda_n = 2L/n \quad n = 1, 2, 3 \dots \text{ (1st harmonic, 2nd harmonic, etc.)} \quad (1)$$

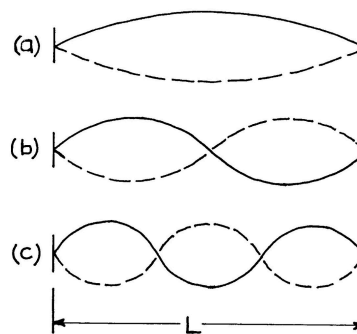


Figure 1: Standing waves on a string.

The **fundamental frequency** (or first harmonic) of vibration of a string is given by

$$f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad (2)$$

where f_1 is the fundamental frequency (1st harmonic, Hz), L is the length of the string (m), T is the tension (N), and μ is the mass per unit length (kg/m).

Possible harmonics are given by

$$f_n = n f_1 \quad n = 1, 2, 3 \dots \text{ (1st harmonic, 2nd harmonic, etc.)} \quad (3)$$

(Example) If one of the variables in Equation 2 is changed, the ratio of the new frequency to the old frequency can be found by writing Equation 2 for each case and taking the ratio of the two. Suppose the length of the string is doubled while the other variables remain fixed, i.e. $T' = T$, $\mu' = \mu$, and $L' = 2L$. The ratio of the new frequency (f') to the old frequency (f) is then given by

$$\frac{f'}{f} = \frac{\frac{1}{2L'} \sqrt{\frac{T'}{\mu'}}}{\frac{1}{2L} \sqrt{\frac{T}{\mu}}} = \frac{\frac{1}{2(2L)} \sqrt{\frac{T}{\mu}}}{\frac{1}{2L} \sqrt{\frac{T}{\mu}}} = \frac{1}{2} \quad \text{after canceling (identical) terms, top and bottom.}$$

The **wave speed**, v , is equal to the product of the frequency, f , and the wavelength, λ , and so

$$v = f_n \lambda_n \quad (4)$$

Frequency of musical notes

An electronic tuning meter, which used to tune musical instruments, is used to measure the frequency of vibration of the strings in this lab experiment. The following table lists the frequencies of the notes which are of interest in this lab.

	Octave			
Note	3	4	5	6
C	130.8	261.6	523.3	1046.5
C#	138.6	277.2	554.4	1108.7
D	146.8	293.7	587.3	1174.7
D#	155.6	311.1	622.3	1244.5
E	164.7	329.6	659.3	1318.5
F	174.6	349.2	698.5	1396.9
F#	185.0	370.0	740.0	1480.0
G	196.0	392.0	784.0	1568.0
G#	207.7	415.3	830.6	1661.2
A	220.0	440.0	880.0	1760.0
A#	233.1	466.2	932.3	1864.7
B	246.9	493.9	987.8	1975.5

DATE:

NAME:
PARTNER:

Laboratory 2: Vibrating Strings

SETUP

1. Place the sonometer (ukelele) with the neck extending slightly past the edge of the bench and the counterweight on the other end of the base.

2. Attach a thin string to the bridge of the sonometer and run it over the pulley at the far end, ensuring the string is properly seated in one of the middle slots at the end of the neck. Attach the 50 g mass hanger to the string and then add 200 g to the hanger.



3. Turn on the tuning meter; it should then be ready for use. DO NOT press buttons unnecessarily. If adjustments are necessary see the appendix at the end of the lab for instructions.

DATA COLLECTION

Part A:

1. [2 marks] Properly measure the freely vibrating length of the string (in cm) & convert to m.

L : _____ - _____ = _____ = _____.

Place the tuning meter close to the hole in the sonometer and pluck the string gently near its centre. Observe the display on the tuning meter. Adjust the mass on the hanger until the string is tuned as closely as possible to note G_3 (“green light”) and record the added mass (in g); convert to kg.
**** NOTE **: masses added to the hanger are good to $\pm 1g$, i.e. $50g = 5.0 \times 10^1 g$.**

m : _____ + 50. g (hanger - see NOTE above!) = _____ = _____.

Record the vibrating string length, total hanging mass, note & octave, frequency (use the table value from the Theory), **harmonic, & waveform sketch** (see Figure 1) **in Table 1.**

Part B: Dependence of frequency on tension (weight)

2. [1 mark] Increase the mass on the hanger until the frequency of the note has doubled from Part A. Record this mass (in g); convert to kg. *Use this mass for Parts B - E, inclusive.* **Record other physical parameters & values as previously in Table 1.**

m : _____ + 50. g (hanger - see NOTE above!) = _____ = _____.

Part C: Generation of harmonics

3. Use the same length L and mass used in Step 2. Generate the *second harmonic* in the following way. Touch the exact centre of the string very lightly with a finger tip. Gently pluck the middle of either string segment with a finger on your other hand, and immediately lift the finger that was touching the centre of the string. If done successfully (it may take several tries) the string will vibrate at twice the fundamental frequency, as in Figure 1b. **Record results in Table 1.**

4. [0.5 mark] Use the same length L and mass used in Step 2. **Predict the note** corresponding to the *third harmonic* of the *fundamental frequency* in Step 2: _____ (HINT: use the definition of harmonics & the table of notes from the Theory section). Generate the *third harmonic* by touching the string at exactly one-third of its length & plucking the middle of the shorter segment. Verify that the note measured by the tuner matches your prediction. **Record results in Table 1.**

Part D: Dependence of frequency on string length

5. [1.5 marks] Use the same mass as in Step 2. *Shorten* the vibrating length L of the string by placing the *moveable bridge* under the string, i.e. between the string and the neck. Pluck the middle of the string segment which is over the sonometer body while gently holding the other segment. Adjust the position of the moveable bridge until the frequency of the vibrations is twice that of step 2. **Measure this new length in cm, convert to m, and record in Table 1.**

L : _____ - _____ = _____ = _____.

What frequency *should* result if the *other* string segment is plucked? Why? (Try it).

Part E: Dependence of frequency on mass per unit length of the string

6. Replace the thinner string with the thicker string. Use the same length L and mass that was used in Step 2. **Record values in Table 1. Tidy the apparatus when you are finished.**

Step	String Size	L (m)	total mass (kg)	note	f_{tuner} (Hz)	harmonic	waveform drawing
1	thin			G3	196.0		
2	thin						
3	thin						
4	thin						
5	thin						
6	thick						

Table 1: Lab data for Steps 1-6 [3 marks].

ANALYSIS

Step	T (N)	λ (m)	v (m/s)	f_{calc} (Hz)	% difference
1					
2					
3					
4					
5					
6					

Table 2: Summary of calculations/analysis [**4 marks**].

1. [**1 mark**] Calculate the tension ($T = mg$) in N for each step and **record in Table 2**. Use $g = 9.81 \text{ m/s}^2$. Show a full sample calculation for Step 2.

2. [**3 marks**] For each step in Table 2 calculate the wavelength λ (Equation 1) and wave speed v (Equation 4 & f_{tuner} from Table 1) and **record in Table 2**. Show sample calculations for Step 2.

Comment on variations in wave speed, in particular with respect to the effect of harmonics.

3. [4 marks] Calculate the frequency f_{calc} for each step using the appropriate values of L , T , μ ($\mu_{thin} = 1.43 \times 10^{-4}$ kg/m, $\mu_{thick} = 5.72 \times 10^{-4}$ kg/m) and Equation 2 (& Equation 3 as required). **Record results in Table 2.** Show sample calculations for *BOTH* Step 2 AND Step 4.

4. [2 marks] Calculate the % difference between frequencies measured by the tuner (f_{tuner}) and your calculated value f_{calc} . **Record results in Table 2.** Show a sample calculation for Step 2.

Comment on your results. If they do *not* agree within 5%, suggest probable experimental causes.

5. [**3 marks**] What value is predicted by theory of vibrating strings for the ratio f_{new}/f_{old} :
- i) when the new string length L is *half* ($1/2$) the original?
 - ii) when the new tension T is *quadruple* (4x) the original?
 - iii) when the new mass per unit length μ is *quadruple* (4x) the original?

Show all steps and use symbols & algebra ONLY! (** see example in Theory section **).

6. [3 marks] Determine the ratio f_{new}/f_{old} *numerically* for EACH of the three cases you examined in part 5. DO NOT recalculate the frequencies - use your previously calculated values of f_{calc} from Table 2 - but MAKE SURE you select the APPROPRIATE pair of frequencies corresponding to each particular case!

For example in part 5i), the case where the new string length is half of the original (while all other physical parameters remain unchanged), the appropriate frequencies would be $f_{new} = f_{calc}$ from Step 5 and $f_{old} = f_{calc}$ from Step 2, yielding $f_{new}/f_{old} = f_5/f_2$. Watch your sig figs/rounding!

Do your *numerical* ratios agree with your *algebraic* predictions from part 5? Comment.

APPENDIX: Operation of the Seiko SAT 500 Chromatic Tuner

1. Press the **POWER** button to turn the instrument on.
2. Press the **SELECT** button (one or more times) to select a menu item from the list on the left side of the screen.
 - a) Select **MODE**. Now use the **UP** and **DOWN** buttons until AUTO appears in the screen (above the **SELECT** button).
 - b) Select **PITCH**. Use the **UP** and **DOWN** buttons until 440 appears in the screen (above the **UP** button). Note that the PITCH must be set to 440 for the notes shown on the meter to correspond to the table of frequencies given in the Theory section.
 - c) Select **KEY** and use the **UP** and **DOWN** buttons until C appears on the screen.
3. The instrument is now ready to use for this lab. The tuner has a battery-saving feature which will cause it to shut off after a pre-determined time. If this happens, press the **POWER** button to turn it back on. The settings that you made will be restored.
4. In order to determine the frequency of vibration of a string, hold the tuning meter close to the hole in the body of the sonometer and pluck the string gently near its centre. On the face of the meter, you will see several indications of the frequency of vibration of the string. All of these work together, as you will see.
 - a) In the lower left corner of the screen, you will see an upper-case letter (which may be followed by the # (sharp) symbol) and a number. The letter is the letter of the musical note which is closest to the one which is being sounded, for example, G. If the sharp symbol is displayed, the note is closer to G#, not G. The number is the octave number in which the note is found. G#4, then, means that the note is G# in the fourth octave.
 - b) In the centre, and occupying most of the screen area, is an indicator needle which gives a visual indication of how far the musical sound is above or below the true value of the note described in the previous step, for example, G#4. If the needle is to the left of centre, the frequency of the string vibration is below the true value of the note and above if the needle is to the right of centre.
 - c) In the lower right corner of the screen, a number (between -50 and +50) indicates the number of cents that the sound is below or above the true value (a cent is 1/100 of the interval between adjacent notes). This number is related to the needle positions described above, with -50 being at the extreme left of the scale, 0 being in the centre and +50 being at the extreme right.
 - d) Finally, there are three LED lights above the screen. The yellow ones to the left and right of centre indicate, respectively, frequencies which are below or above the true indicated value. When the sound matches the displayed note (G#4, for example) the green LED lights.

