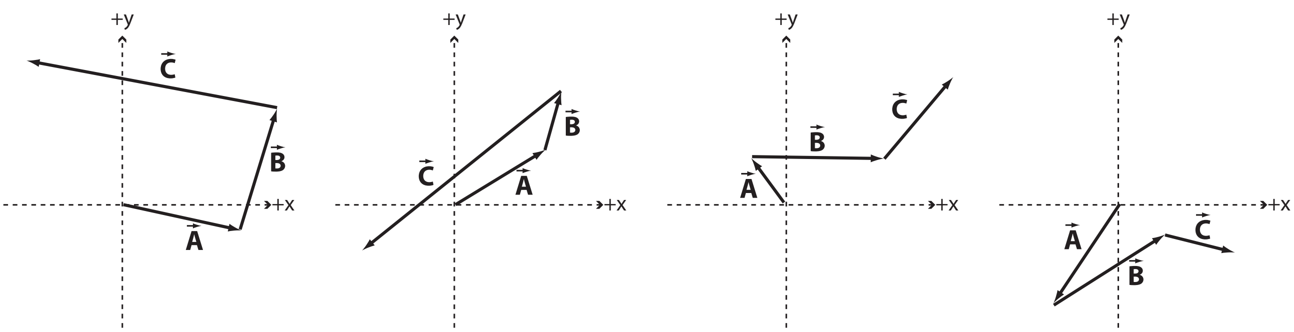
Laboratory 2 Pre-Lab (value: *2 marks*)

Submit to your lab instructor ***by 4pm the day BEFORE*** your lab period.

1. What condition(s) must be met by an object which is in *static equilibrium*?

2. Draw $\vec{\mathbf{D}}$ that 'closes' the polygons shown & measure the polar angle θ, CCW from the +x-axis:



(a) (b) (c) (d)

3. Calculate the x and y components of a force of 11.6 N at 171°. Show all work & watch signs!

4. Put F_x = -11.5 N, F_y = 1.81 N into *magnitude & direction* form, with the angle CCW to +x-axis.

Laboratory 2:
Applications of Static Equilibrium to Models

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Arm model, bench clamp, stand, two 40 cm rods, cross clamp, short rod, set of hooked masses, set of slotted masses, set of kilogram masses (1.000kg, 0.500kg, 2.000kg), large mass hanger, metre stick and support clamp with black pointer, short wooden metre stick with calipers, metric 1cm grid graph paper. Students typically supply their own 30 cm plastic ruler, protractor.

OBJECTIVE

1. To apply the principles of static equilibrium to a model of a human arm.
2. To determine the reaction force at the elbow joint of a loaded arm.

THEORY

Models are frequently used as an aid in studying the properties of physical systems. Since real systems are typically quite complex, all models involve some degree of simplification and approximation. However, even a relatively simple model can be useful in providing insight into the characteristics of a real system. In this experiment, wooden pieces joined with bearings are used to model the behaviour of a human arm under load. The biceps muscle is modeled by a tension spring. The magnitude and direction of the force on the elbow joint are found by applying the conditions for static equilibrium.

Static equilibrium in two dimensions

According to Newton's second law, the two conditions for static equilibrium require that

- the vector sum of the external forces acting on the body is zero, and
- the vector sum of the external torques acting on the body is zero.

In this lab we study the first condition for equilibrium; in the next lab we examine the second.

Graphical addition of vectors

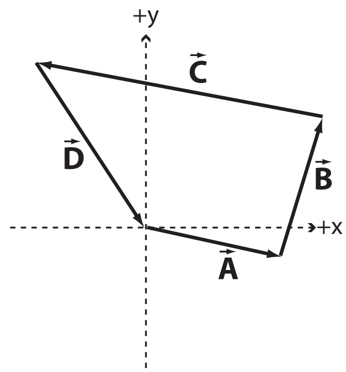


Figure 1: Vector polygon

In the *polygon method* of vector addition, scaled vectors (whose measured lengths correspond to the magnitudes of the forces that they represent, e.g. 1 cm = 3 N) are drawn such that each vector starts at the end point of the previous one and points in the same direction as the force it represents. In Figure 1, suppose that the vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ are all of the forces acting on an object in static equilibrium. The sum of these forces *must* be zero, as shown in the figure (i.e. adding these vectors in any order produces a *closed* polygon). Any number of vectors can be added head-to-tail in this way; if the object is in equilibrium, the forces MUST sum to zero, regardless.

Analytical addition of vectors

Components of a vector

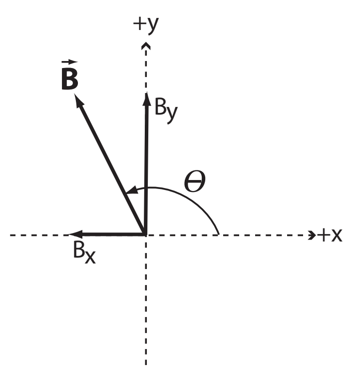


Figure 2: Vector components

The vector $\vec{\mathbf{B}}$ and a set of orthogonal axes (x & y) are shown in Figure 2. The components, B_x and B_y, are the projections of $\vec{\mathbf{B}}$ on the two axes. The components are given by

$$B_x = B\cos\theta$$
$$B_y = B\sin\theta$$

By convention, the **polar angle** θ is measured **counterclockwise from the positive x-axis** to the arrow end of the vector, and has a value between 0 and 360 degrees. If this convention is followed, the algebraic sign of the trigonometric function will automatically indicate whether a component is positive or negative (i.e. you do not need to manually 'add' the sign).

Resultant, magnitude and direction

If the components of a vector are known, the *magnitude and direction* of the vector are found by

$$|\vec{\mathbf{B}}| = \sqrt{B_x^2 + B_y^2} \tag{1}$$

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) \tag{2}$$

Note that your calculator will always yield a value of θ that places the vector in Quadrant I or IV; you must, on the basis of the *signs* of B_x and B_y, determine in which quadrant the vector *actually resides* and *potentially* add 180° or 360° to the calculator's value for θ, as appropriate.

Addition of components

The x and y components of the sum of the addition of several vectors are, respectively, the sum of the x components of the individual vectors and the sum of the y components of the individual vectors. For example, if $\sum \vec{\mathbf{F}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$, then

$$\sum F_x = A_x + B_x + C_x + D_x$$

$$\sum F_y = A_y + B_y + C_y + D_y$$

where the x and y components for each vector are found as shown previously. From these sums the magnitude and direction of the resultant or net force can then be found. ***If an object is in equilibrium then*** $\sum F_x = \sum F_y = 0$.

Arm model

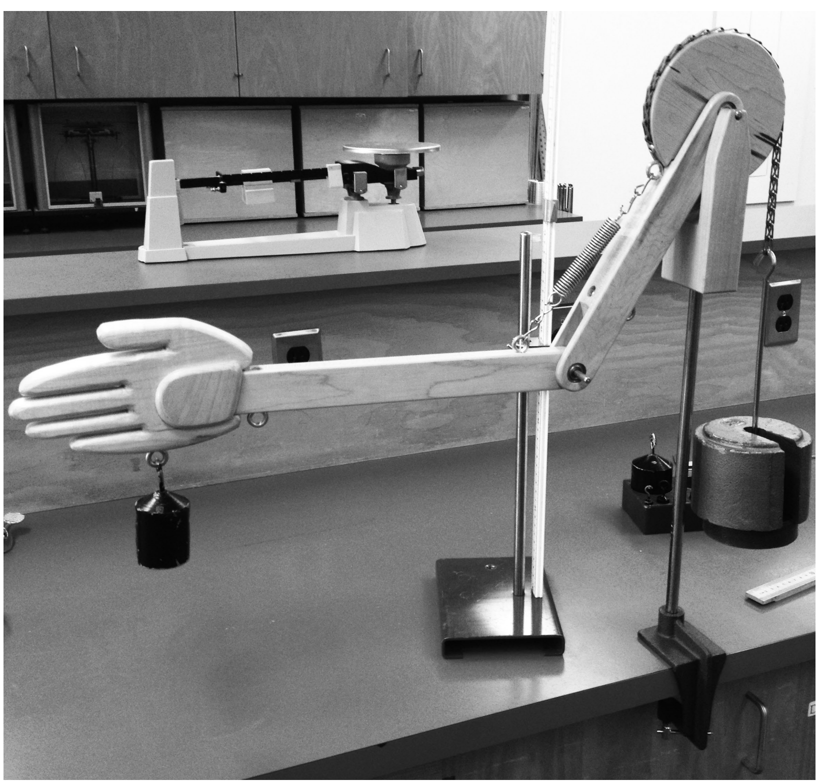


Figure 3: The Arm model

NAME:
DATE: PARTNER:

Laboratory 2: Applications of Static Equilibrium to Models

Setup and Measurements

1. Set up the arm as directed (see Figure 3), **MAKING SURE the hand 'points' to the LEFT**.

2. Hang hooked masses totaling 700. g on the hand ($m_{load\,on\,hand}$) THEN place a counterweight of ∼5.000 kg on the hanger hooked to the vertical shoulder chain; the hanger itself has a mass of 1.000 kg. *The lower arm should be ∼ horizontal & the upper arm should NOT touch the shoulder support*; adjust the apparatus by adding more mass to the hand and/or CW as required. Displace the hand slightly from equilibrium; the system should oscillate freely and return to rest.

3. [***2 marks***] **Draw a FULLY LABELLED view of the apparatus** below. Record the mass added to the hand (eg. $m_{load\,on\,hand} = 700.\,\text{g}$) and counterweight hanger, L & Δy, etc.

4. [***1 mark***] Mount the calipers on the wooden metre stick, spaced so that they just fit over the **OUTERMOST loops of the STRETCHED biceps spring**. Lock them in place and **LEAVE the calipers set at this length for the REMAINDER OF THE LAB.** Record the positions of the calipers on the metre stick (**in cm**) to measure the length, L, of the stretched spring:

$L = L_f - L_i =$ ______________________________ = ____________.

5. [***1 mark***] Mount the plastic metre stick vertically on the support stand and use the sliding black pointer to measure (**in cm**) the VERTICAL positions (y-coordinates) of the **OUTERMOST** loops of the stretched bicep spring (i.e. the same points used in measuring L above):

$\Delta y = y_f - y_i =$ ______________________________ = ____________.

6. Remove the large mass from the hanger, THEN the mass on the hand; dismantle the arm model.

7. [***1 mark***] **Measure** the combined mass of the lower arm & hand (**in g**); convert to kg.

$m_{arm+hand}$ =______________________________ = _______________.

8. [***1 mark***] Mount the cross bar FIRMLY at the top of the support rod and hang the bicep spring from it. Attach the 1.000 kg mass hanger DIRECTLY to the lower end of the spring and add mass (start with several kilograms and then use smaller masses) to the hanger until the spring is stretched to the SAME length L as on the arm model (use the UNALTERED spacing of the calipers on the wooden ruler for comparison/verification). **Record** the total mass required to achieve this ($m_{stretch}$ *may* be similar to the counterweight mass but it is NOT necessarily identical to it):

$m_{stretch}$ =______________________ + 1.000 kg (hanger) = _______________.

9. Tidy the apparatus.

Analysis and Results

**** Assume the gravitational acceleration to be $g = 9.81$ m/s^2 and to have 3 sig figs. ****

1. [***1 mark***] Calculate $F_g = mg = (m_{arm+hand} + m_{load\,on\,hand})g$, the (total) *weight* of the lower arm, hand, and the mass added to the hand. *Watch your units & show ALL work.*

2. [***1 mark***] Calculate the magnitude of the biceps muscle force, $F_b = mg$, using the mass $m_{stretch}$ required to stretch the spring. *Watch your units & show ALL work.*

3. [***1 mark***] Calculate the angle that the spring makes with the horizontal, $\theta_b = \sin^{-1}(\Delta y/L)$, using the previously measured length L of the stretched spring & the height difference Δy between the ends of the spring; θ_b is also the *direction of the bicep force*, $\vec{\mathbf{F}}_b$. *Show ALL work.*

4. [***3 marks***] Draw a **FREE-BODY DIAGRAM** (FBD) for the lower arm & hand IN THE SPACE BELOW. The free-body diagram MUST have the same orientation as your arm model setup/drawing from earlier; DO NOT flip your FBD left-to-right. Draw and label the forces ($\vec{\mathbf{F}}_g$, $\vec{\mathbf{F}}_b$, $\vec{\mathbf{F}}_e$) acting on the system, including the angles at which they act (*angles should be referenced using the standard convention, i.e. counterclockwise from the positive x-axis*). **Magnitudes & directions for the forces are only meant to be approximate, NOT exact**. Label your FBD's vectors using SYMBOLS ONLY (e.g. $\vec{\mathbf{F}}_b$, θ_b), NOT numbers. IN WORDS explain how you arrived at the (approximate) magnitude & direction of the unknown force $\vec{\mathbf{F}}_e$.

5. [***6 marks***] This section utilizes the graphical or 'vector polygon' method to determine the (unknown) elbow force. **ON THE SEPARATELY PROVIDED GRAPH PAPER, draw a vector polygon of the known forces (bicep, gravitational) acting on the lower arm to scale**. Use a sharp pencil, ruler and protractor when drawing vectors. ***Label* all forces, angles, the x & y coordinate directions, and the scale used**, *eg. 1 cm = 2.5 N or 1 cm = 3 N.* Choose a (simple) scale that maximizes the size of the vectors but still fits comfortably on the page.

Once all known forces are drawn 'tip-to-tail', **draw the vector which must be added to the others in order to 'close' the polygon** (*i.e. result in a net force of zero*). This vector represents the (unknown) elbow force $\vec{\mathbf{F}}_e$ which balances the other (known) forces. **Measure** the length of the elbow force on your diagram (in cm) & use your scale to convert this length to the **magnitude of the elbow force** (in newtons). **Measure** the angle that the elbow force makes *counterclockwise from the positive x-axis* to determine the **direction of the elbow force**. **Label** both the magnitude and the direction/angle of the elbow force on your diagram.

Show ALL scaling calculations *neatly* ON THE GRAPH PAPER; watch your sig figs!.

6. The following questions use the analytical (vector components) method to determine (the unknown force) $\vec{\mathbf{F}}_e$. Please note that this method is **INDEPENDENT** of your vector polygon so DO NOT use ANY values obtained from your drawing in the steps below. At the end of the lab you will COMPARE the results of the two approaches used to find $\vec{\mathbf{F}}_e$ and see how well they agree.

7. [***2 marks***] Referring to your free-body diagram & USING ONLY SYMBOLS, **write the equilibrium equations** for the sum of the forces in the x-direction AND for the sum of the forces in the y-direction. Recall that for an object in *equilibrium*, $\sum F_x = \sum F_y = 0$. SOLVE the equations *algebraically* for the (unknown) components of the elbow force, F_{ex} and F_{ey}, eg. $F_{ex} = \ldots$ and $F_{ey} = \ldots$.

DO NOT INSERT NUMBERS! *Watch signs & show ALL steps.* For example, in the x-direction:

$$\begin{aligned}\sum F_x &= F_{gx} + F_{bx} + F_{ex} \;=\; 0 \\ 0 &= 0 + F_b \cos\theta_b + F_{ex} \\ F_{ex} &= -F_b \cos\theta_b\end{aligned}$$

Using SYMBOLS ONLY, derive the corresponding equation for F_{ey} in the y-direction:

8. [***3 marks***] **Calculate** F_{ex} & F_{ey} using the above equations, substituting (unrounded) values for F_g, F_b & θ_b from earlier in the lab (NOT the vector polygon!). *Watch signs & show ALL steps.*

9. [***2 marks***] **Calculate** the *magnitude & direction* of the elbow force using F_{ex} & F_{ey} (above) and Equations 1 & 2. Reference the direction *as an angle CCW from the +x-axis. Show ALL work.*

10. [***3 marks***] **Compare** the magnitudes ($|\vec{\mathbf{F}}_e|$) found using the graphical & analytical methods using *percent difference* (see Lab 1); **repeat** the comparison for the polar angles (θ_e). **Comment** on your comparisons & their agreement. **Which method *should* be more accurate? Explain**.