Laboratory 2 Pre-Lab (value: *2 marks*)

Submit to your lab instructor ***by 4pm the day BEFORE*** your lab period.

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1. What musical note corresponds to a frequency of 220 Hz? Which note would be its 3rd harmonic?

2. Which harmonic is shown in Fig 1c? Calculate the wavelength λ of this wave if $L = 60.\,\mathrm{cm}$.

3. Referring to Equation 2, does frequency go *up* or *down* when: (a) string length (only) is doubled? (b) mass-per-unit length (only) is halved? (c) string length (only) is halved? (d) tension is tripled *and* string length is doubled? (e) tension, string length and mass-per-unit length are *all* doubled?

4. The wave speed relationship of Equation 4 applies to all harmonics, not only the fundamental frequency of vibration. *Using symbols*, substitute for λ_n and f_n in Equation 4 using Equations 1, 2 & 3, and then simplify the result *algebraically*, showing your work/steps. Does your final answer for the wave speed depend on n? What does this imply regarding wave speed and harmonics?

Laboratory 2: Vibrating Strings

Experiments are to be completed on the provided laboratory sheets below; any supporting material (eg. graphs) should be attached. Make sure your name and your partners name(s) are clearly indicated on the front page of your lab. **Neatness and clarity count!** Explain your answers clearly and concisely. If an equation is to be used in a calculation, *write the equation down* and then insert numbers and solve. Report your final answer to the appropriate significant figures.

The lab write-up is due by the end of the lab. Late labs will not be accepted.

APPARATUS

Mounted ukulele sonometer, moveable bridge, 5 kg counterweight, 2 strings ($\mu_1 = 1.43 \times 10^{-4}$ kg/m, $\mu_2 = 5.72 \times 10^{-4}$ kg/m), mass hanger, 2 sets of slotted masses (2000g total, including 5 and 2 g masses), half-metre stick, tuning meter, pulley with stand setup.

OBJECTIVE

1. To study the properties of standing waves on strings.
2. To find the velocity of a wave traveling along a string.

THEORY

When a string is set into vibration by plucking it standing waves can be generated. Examples of standing waves for a stretched string of length L are shown in Figure 1. The lowest possible frequency (the **fundamental frequency**) is produced when the string vibrates in one segment, as in Figure 1a. The string in Figure 1b is vibrating in its **second harmonic** mode (twice the fundamental frequency). Figure 1c shows the **third harmonic** (three times the fundamental frequency).

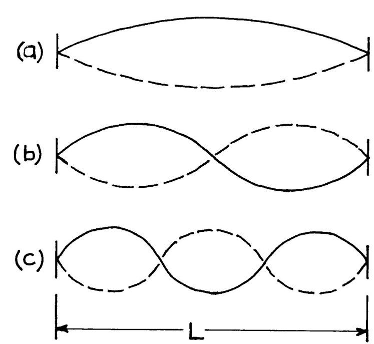

Figure 1: Standing waves on a string.

The wavelength of the generated waves is equal to twice the nodal separation (positions of zero amplitude); hence, the wavelength corresponding to the n^{th} harmonic is given by

$$\lambda_n = 2L/n \qquad n = 1, 2, 3\ldots \text{ (1st harmonic, 2nd harmonic, etc.)} \qquad (1)$$

The **fundamental frequency** (or first harmonic) of vibration of a string is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{2}$$

where f_1 is the fundamental frequency (1st harmonic, Hz), L is the length of the string (m), T is the tension (N), and μ is the mass per unit length (kg/m).

Possible harmonics are given by

$$f_n = n\, f_1 \qquad n = 1, 2, 3\ldots \text{ (1st harmonic, 2nd harmonic, etc.)} \tag{3}$$

(Example) *If one of the variables in Equation 2 is changed, the ratio of the new frequency to the old frequency can be found by writing Equation 2 for each case and taking the ratio of the two. Suppose the length of the string is doubled while the other variables remain fixed, i.e.* $T' = T$, $\mu' = \mu$, *and* $L' = 2L$. *The ratio of the new frequency* (f') *to the old frequency* (f) *is then given by*

$$\frac{f'}{f} = \frac{\frac{1}{2L'}\sqrt{\frac{T'}{\mu'}}}{\frac{1}{2L}\sqrt{\frac{T}{\mu}}} = \frac{\frac{1}{2(2L)}\sqrt{\frac{T}{\mu}}}{\frac{1}{2L}\sqrt{\frac{T}{\mu}}} = \frac{1}{2} \qquad \textit{after canceling (identical) terms, top and bottom.}$$

The **wave speed**, v, is equal to the product of the frequency, f, and the wavelength, λ, and so

$$v = f_n\, \lambda_n \tag{4}$$

Frequency of musical notes

An electronic tuning meter, which used to tune musical instruments, is used to measure the frequency of vibration of the strings in this lab experiment. The following table lists the frequencies of the notes which are of interest in this lab.

	Octave			
Note	3	4	5	6
C	130.8	261.6	523.3	1046.5
C#	138.6	277.2	554.4	1108.7
D	146.8	293.7	587.3	1174.7
D#	155.6	311.1	622.3	1244.5
E	164.7	329.6	659.3	1318.5
F	174.6	349.2	698.5	1396.9
F#	185.0	370.0	740.0	1480 .0
G	196.0	392.0	784.0	1568.0
G#	207.7	415.3	830.6	1661.2
A	220.0	440.0	880.0	1760.0
A#	233.1	466.2	932.3	1864.7
B	246.9	493.9	987.8	1975.5

Laboratory 2:
Vibrating Strings

SETUP

1. Place the sonometer (ukelele) with the neck extending slightly past the edge of the bench and the counterweight on the other end of the base.

2. Attach the thin string to the bridge of the sonometer and run it over the pulley at the far end, ensuring the string is properly seated in one of the middle slots at the end of the neck. Attach the 50. g mass hanger to the string and add 200. g to the hanger.

3. Turn on the tuning meter; it should then be ready for use. DO NOT press buttons unnecessarily! If adjustments to the settings are necessary please speak with your instructor.

DATA COLLECTION

Part A:

1. [***2 marks***] **Measure the freely vibrating length of the string** (in cm) & convert to m.

L : __________ - __________ = _____________ = _____________.

Place the tuning meter close to the hole in the sonometer and pluck the string gently near its centre. Observe the display on the tuning meter. Adjust the mass on the hanger until the string is tuned as closely as possible to note G_3 ("green light") and record the added mass (in g); convert to kg. **** NOTE **: masses added to the hanger are good to ± 1 g, i.e. 50. g = $5.0\text{x}10^1$ g.**

m : __________ + 50. g (hanger - see NOTE above!) = _____________ = _____________.

Record the vibrating string length, total hanging mass, note & octave, frequency (use the table value from the Theory), **harmonic, & waveform sketch** (see Figure 1) **in Table 1**.

Part B: Dependence of frequency on tension (weight)

2. [***1 mark***] Increase the mass on the hanger until the frequency of the note has doubled from Part A. Record this mass (in g); convert to kg. *Use this mass for Parts B - E, inclusive.* **Record other physical parameters & observations in Table 1**.

m : __________ + 50. g (hanger - see NOTE above!) = _____________ = _____________.

Part C: Generation of harmonics

3. Use the same length L and mass used in Step 2. Generate the *second harmonic* in the following way. Touch the exact centre of the string very lightly with a finger tip. Gently pluck the middle of either string segment with a finger on your other hand, and immediately lift the finger that was touching the centre of the string. If done successfully (it may take several tries) the string will vibrate at twice the fundamental frequency, as in Figure 1b. **Record results in Table 1**.

4. [***0.5 mark***] Use the same length L and mass used in Step 2. **Predict the note** corresponding to the *third harmonic* of the *fundamental frequency* in Step 2: ________ [*HINT: use the definition of harmonics & the table of notes from the Theory section*]. Generate the *third harmonic* by touching the string at exactly one-third of its length & plucking the middle of the shorter segment. Verify that the note measured by the tuner matches your prediction. **Record results in Table 1**.

Part D: Dependence of frequency on string length

5. [***1.5 marks***] Use the same mass as in Step 2. *Shorten* the vibrating length L of the string by placing the *moveable bridge* under the string, i.e. between the string and the neck. Pluck the middle of the string segment which is over the sonometer body while gently holding the other segment. Adjust the position of the moveable bridge until the frequency of the vibrations is twice that of step 2. **Measure this new length in cm, convert to m, and record in Table 1**.

L : __________ - __________ = ______________ = ______________.

What frequency *should* result if the *other* string segment is plucked? Why? (Try it).

Part E: Dependence of frequency on mass per unit length of the string

6. Replace the thinner string with the thicker string. Use the same length L and mass that was used in Step 2. **Record values in Table 1**. **Tidy the apparatus when you are finished.**

Step	String Size	L (m)	total mass (kg)	note	f_{tuner} (Hz)	harmonic	waveform drawing
1	thin			G3	196.0		
2	thin						
3	thin						
4	thin						
5	thin						
6	thick						

Table 1: Lab data for Steps 1-6 [***3 marks***].

ANALYSIS

Step	T (N)	λ (m)	v (m/s)	f_{calc} (Hz)	% difference
1					
2					
3					
4					
5					
6					

Table 2: Summary of calculations/analysis [***4 marks***].

1. [***1 mark***] **Calculate the tension** ($T = mg$) in *newtons* (N) for each step; use $g = 9.81$ m/s^2. **Show a full sample calculation for Step 2. Record all results in Table 2**.

2. [***3 marks***] **Calculate wavelength** λ (Equation 1) **& wave speed** v (Equation 4 & f_{tuner} from Table 1) for each step. **Show sample calculations for Step 2. Record results in Table 2**.

Comment on variations in wave speed, in particular with respect to the effect of harmonics.

3. [*4 marks*] **Calculate the frequency** f_{calc} for each step using Equation(s) 2 & 3 and appropriate values of L, T, and μ. **Show sample calculations for *BOTH* Step 2 AND Step 4**. [*NOTE*: $\mu_{thin} = 1.43 \times 10^{-4}$ kg/m, $\mu_{thick} = 5.72 \times 10^{-4}$ kg/m] **Record all results in Table 2**.

4. [*2 marks*] **Compare** measured frequencies (f_{tuner}) with calculated values f_{calc} for each step using *percent difference*. **Show a sample calculation for Step 2. Record results in Table 2**. **Comment** on your results. If they do *not* agree within 5%, suggest probable experimental causes.

5. [***3 marks***] What does Equation 2 predict for the ratio of frequencies f_{new}/f_{old} when:

   i) the new string length L is *half* ($^1/_2$) the original length?

   ii) the new tension T is *quadruple* (4x) the original tension?

   iii) the new mass per unit length μ is *quadruple* (4x) the original μ?

Show all steps & use symbols/algebra ONLY! (** see example in the Theory section **).

[*NOTE: in each case,* ONLY *the specified variable is changed - all other variables remain the same.*]

6. [***3 marks***] **Verify your algebraic predictions for the ratio** f_{new}/f_{old} in parts 5(i) - 5(iii) ***numerically*** using appropriate *pairs* of frequencies from your **previously calculated values of** f_{calc} **in Table 2. ** DO NOT recalculate ANY frequencies! ****

(Example) Part 5(i), where the new string length is half the original *while all other physical parameters remain unchanged*, corresponds to $f_{new} = f_{calc}$ (Step 5) versus $f_{old} = f_{calc}$ (Step 2); in this case, $f_{new}/f_{old} = f_{calc\,5}/f_{calc\,2}$, calculated using your values of $f_{calc\,5}$ and $f_{calc\,2}$ from Table 2.

Use the properly corresponding pairs of frequencies for EACH part & watch your sig figs/rounding!

Do your *numerical* ratios agree with your *algebraic* predictions from part 5? Comment.